Modelling the Spring Constant with the Classical Work-Energy Theorem
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Introduction

The aim of the egg bungee challenge is to drop an egg from a set height and have it get as close to the ground without breaking the egg. The egg will be held only by a bungee cord with spring-like characteristics. The experiment considers many dynamic and static conditions and in a previous experiment, we investigated static conditions by varying lengths of elastic string. The elastic string can be modelled to represent Hooke’s Law (Equation 1) and we determined an expression to calculate the k value based off the length of chord.

\[ F = -kx \]  
(Equation 1)

In order to expand our understanding of the elastic cord as an effective spring, we evaluated the k value in a dynamic system using the Classical Work Energy theorem (Equation 2). When we assume there are no other forces acting on the system, \( W_{\text{other}} = 0 \) and therefore, we can take measurements where there is no kinetic energy (the system at the top at rest and the system at the lowest point of the fall). This derivation suggests that the initial potential energy and the final potential energy are conserved, and the respective potential energy formulas can be substituted (Equation 3). Based off this equation, we could calculate k by measuring starting height and the displacement from which the mass falls and compare the k values of various lengths of string. By comparing length to k value, we can create another expression that correlates length of cord and k value and we can use that expression to calculate the k for any length of cord.

\[ W_{\text{other}} = \Delta(KE + PE) \]  
(Equation 2)

\[ W_{\text{other}} = (KE + PE)_f - (KE + PE)_i \]

\[ 0 = (KE + PE)_f - (KE + PE)_i \]

\[ 0 = PE_f - PE_i \]

\[ PE_i = PE_f \]

\[ mgh = (1/2)kx_i^2 \]  
(Equation 3)

The purpose of this experiment is to investigate the k value of an elastic string in a dynamic system. Can a dynamic system better represent the spring constant of an elastic string than in a static system? With our determined k value expression, we hope to better understand the nature of elastic strings and to discuss any differences in our previous experiment and with others who are a part of the bungee challenge. We hypothesized that as the length of string increased, the
spring constant would decrease as longer strings have a tendency to be stretched further than short strings.

**Methods**

We varied only the lengths of elastic string and kept all other variables the same. We set up the system on a metal stand tightly bound to a level lab table. Figure 1 sums up the setup for the cord system we used. The mass was held constant at 0.1505 kg ± 0.0001 kg and the weights were held together by blue masking tape. The weight was also taped to the cord to keep the weight from constantly slipping out of the loop made in the string. The string was looped on either end for the weight and stand to hook through. A tape measurer was hung and taped from the stand’s arm for all measurements.

![Figure 1](image.png)

**Figure 1.** This figure shows the general setup of the system. This picture displays the weight in equilibrium position, where the weight is hanging without movement. We measured equilibrium position for each trial and length of string. The weight was dropped from the top and the maximum length that the weight reached was recorded as the height. $\Delta x$ was calculated by subtracting the equilibrium position length from the height.

To get an accurate k value for each length, we measured the length of the unstretched string, the cord length in equilibrium (Figure 1), and the maximum length of stretch during the fall. This maximum length represents the height of the system and the displacement, $x$, could be calculated by subtracting the equilibrium position from the maximum length of stretch. The bottom of the weight was dropped from where the top loop met the stand and stretch lengths were measured using a slow-motion camera. We measured from the bottom of the weight and subtracted the length of the weight (0.19 m ± 0.01 m) from the maximum height to get an accurate measure of the string’s stretched length.
We tested four lengths of string and ran five trials for each length. We solved for $k$ for each respective value by adjusting Equation 3 with respect to $k$:

$$k = 2mg(x^2)$$

In order to achieve our expression for the spring constant, we averaged the $k$ values for each length and plotted the $k$ value with respect to the unstretched string length. Uncertainties were calculated from standard deviations of data sets except with the linearized data, where uncertainties were calculated manually. We then linearized the graph to attain a more accurate trend line with the data and the equation of the trend line is the expression to find the spring constant for any length of string.

**Results**

We calculated average $k$ values for each height (Table 1). As length increases, the value of the spring constant decreases. We then plotted this data onto a graph (Figure 2) with length on the x-axis and spring constant on the y-axis.

<table>
<thead>
<tr>
<th>Length (m) (± 0.01 m)</th>
<th>Average k (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.255</td>
<td>8.88 ± 0.7</td>
</tr>
<tr>
<td>0.345</td>
<td>6.95 ± 0.1</td>
</tr>
<tr>
<td>0.41</td>
<td>6.23 ± 0.02</td>
</tr>
<tr>
<td>0.47</td>
<td>5.49 ± 0.1</td>
</tr>
</tbody>
</table>

**Table 1.** Comparing the length of string with calculated average $k$ value. Measurements were made in cm but converted to meters.

In Microsoft Excel, the trend line which best fit the data points was an exponential function. However, while the exponential line gave a better fit, calculations become more complicated with a power graph. We linearized the data by plugging in lengths at $L^{-0.773}$ (Table 2).

![Figure 2](image.png)

**Figure 2.** The graph shows a preliminary attempt at creating an expression for a spring constant. To create a more convenient expression, this graph was linearized. In the expression, $L$ is the length.
The data was then plotted onto a new graph with $L^{-0.773}$ on the x-axis and average $k$ on the y-axis (Figure 3). The graph gives us the desired expression with which we can calculate the spring constant with any length of unstretched string (Equation 4).

\[ k = 3.0841L \]  \hspace{1cm} \text{(Equation 4)}

<table>
<thead>
<tr>
<th>Length$^{-0.773}$ (m$^{-0.773}$)</th>
<th>Average $k$ (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.88 ± 0.02</td>
<td>8.88 ± 0.7</td>
</tr>
<tr>
<td>2.27 ± 0.02</td>
<td>6.95 ± 0.1</td>
</tr>
<tr>
<td>1.99 ± 0.02</td>
<td>6.23 ± 0.02</td>
</tr>
<tr>
<td>1.79 ± 0.01</td>
<td>5.49 ± 0.1</td>
</tr>
</tbody>
</table>

Table 2. Original lengths were powered to a magnitude of -0.773. Uncertainties vary for each length of string because there is a dependence on the ratio of the new uncertainty and the new calculated value, R.

Figure 3. The graph and trend line for our spring constant expression. The equation calculates the spring constant at any length of the specific bungee cord we are using for the bungee drop.

Discussion

Our results suggest an exponential relationship between cord length and spring constant. We linearized this exponential trend and created a simpler expression. We hoped to find a perfectly linear expression, however, many factors might have contributed to an exponential trend. This was similar to our previous static experiment where we also had data best fit an exponential trend line. Based off this similarity, we expected that the linearization would give a similar linear graph.
In comparing our dynamic data to our static data, the slope of the current expression is more than double the slope of the expression derived from our static data. Of course, there is no reference for which is the more accurate expression, but it can be inferred that the static expression might be more accurate because it is a simpler experiment. The current experiment had many potential experimental errors factor into the uncertainties. For example, it was difficult to get precise data on the shorter length of string as it fell more sporadically than the longer lengths. This is evident in the uncertainty of the average k value for the shortest string (Table 1). Regarding the longer lengths, it was more likely for the weight to ‘wobble’ during the fall. We noted that a wobble typically resulted in shorter displacement. We assumed that our system was a part of a friction-less system. Friction, in the form of air resistance, might have played a role in not allowing the mass to stretch as far as it could.

If the experiment were repeated, a wider range of lengths of string would be used. Space was limited in the lab and lengths of string exceeding our longest string resulted in the weight impacting the floor. We would also consider using separate pieces of string to combat any prolonged stretch effects the string might have.

Conclusion

We hypothesized that as the length of string used increased, the spring constant would decrease. While our hypothesis was correct, there was great variation between our previous expression relating length of string and spring constant. We believe that the previous expression, derived from a static experiment, would be a better candidate for accurately quantifying the spring constant because this dynamic experiment had a larger perceived margin of error.

For the final egg drop experiment, we might lean towards using the spring constant derived from our static experiment. There is no quantifiable evidence on that decision, but it can be inferred that the static experiment was more simply set up and therefore more accurate.