The jump is coming: Using Hooke's Law to describe bungee cord motion

Introduction

The purpose of this lab is to investigate how to most accurately model the behavior of the bungee cord provided. As its behavior appears to be somewhat similar to that of a spring, Hooke's law may provide a useful base for our investigation. The law is described by the following equation:

\[ F_{\text{spring}} = -kx \]

Where \( k \) is a spring constant and \( x \) is the displacement of the spring in response to \( F \). \( K \) can be determined experimentally by using Newton's second law to define \( F_{\text{spring}} \):

\[ F_{\text{spring}} = mg \]

Where \( m \) is the mass applied to the cord. In two previous experiments, we measured displacement at different masses for a set cord length. Our experiment found the relationship between Force and displacement to be better modeled by the following power equation:

\[ F = -kx^{3/4} \]

We will proceed in this experiment using this equation to describe the motion of the bungee cord. We also found experimentally that \( k \) is different at different cord lengths. In other words, using a longer length of the same cord will constitute an entirely different spring. This is consistent with our knowledge that \( k \) is a constant specific to a particular spring.

In this experiment, we sought to further model the behavior of the spring and increase our understanding of its motion. We will primarily focus on better understanding the effect of cord length on \( k \). These findings will critically impact our design for a successful egg bungee jump that will occur at the end of the term.

Methods

To further our understanding of the behavior of the cord, we measured how displacement varies based on cord length. This will also enable us to determine how stretch (\( k \)) changes due to increasing cord length.
To do this, we kept mass constant at 0.150 kg by hanging 0.100 kg from the 0.050 kg hangar. We then measured displacement at 9 cord lengths between 0.11 m and 1.33 m. Measurements were taken from the point that the top of the knotted loop intersects with the knob to the top of the hangar. Knot loops were made as small as possible to decrease the effect of the short segment of double cord. All measurements were taken in meters using a measuring tape. Each mass was allowed to fully equilibrate before measurement was taken. Displacement was calculated by subtracting the final cord length from the initial. The k for each cord length was then calculated by applying Equation 1. Fig. 1 diagrams the experimental set up.

**Fig. 1: Set Up.** Materials used for determination of displacement.

Results

Using the methods described above, we collected the data in Table 1. Using Equation 2, we found that weight, or Force, is 1.47 N. From this, $x^{3/4}$ was calculated according to Equation 3. The value of $k_{corrected}$, or $k_c$, was calculated using $x^{3/4}$ instead of $x$. Uncertainties of calculated values $x^{3/4}$ and $k_c$ were determined using standard propagation of uncertainty.

**Table 1: Displacement of mass at varying cord length.** Displacement measured at each cord length using a 0.150 kg mass.

<table>
<thead>
<tr>
<th>cord length, l (m, ± 0.01 m)</th>
<th>displacement, x (m, ± 0.01 m)</th>
<th>$x^{3/4}$</th>
<th>percent uncertainty, $x^{3/4}$ (%)</th>
<th>k (N/m)</th>
<th>uncertainty, k (m)</th>
<th>k corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.11</td>
<td>0.22</td>
<td>6</td>
<td>11.31</td>
<td>0.08</td>
<td>6.79</td>
</tr>
<tr>
<td>2</td>
<td>0.23</td>
<td>0.29</td>
<td>5</td>
<td>7.74</td>
<td>0.05</td>
<td>5.11</td>
</tr>
<tr>
<td>3</td>
<td>0.29</td>
<td>0.33</td>
<td>4</td>
<td>6.39</td>
<td>0.04</td>
<td>4.43</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>0.38</td>
<td>4</td>
<td>5.44</td>
<td>0.04</td>
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</tr>
<tr>
<td>5</td>
<td>0.36</td>
<td>0.42</td>
<td>3</td>
<td>4.74</td>
<td>0.03</td>
<td>3.54</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
<td>0.49</td>
<td>3</td>
<td>3.77</td>
<td>0.03</td>
<td>2.98</td>
</tr>
</tbody>
</table>
Using this data we plotted displacement, $x$ vs. cord length, $l$, shown in Fig. 4.

**Fig. 4: Displacement vs. cord length.** Displacement due to a 0.150 kg mass at 9 cord lengths.

![Displacement vs. cord length](image)

The relationship shown in the graph in Fig. 4 is described by the following linear trendline:

Equation 4: $x = 0.89l - 0.005$

Here we see that as resting cord length increases, displacement also directly increases. Using linear regression, we determined the standard error of the slope to be 0.03, and the standard error of the y-intercept as 0.02.

We then determined $k_c$ for each cord length by rearranging Equation 3:

Equation 5: $k_c = \frac{F}{x^{3/4}}$

We used the calculations of $k_c$ to graph $k_c$ vs. $\frac{1}{\text{cord length}}$, shown in Fig. 5. The calculated $\frac{1}{\text{cord length}}$ was used to show a positive relationship between the two variables.
Fig. 5: \( k_c \) vs. \( \frac{1}{\text{cord length}} \). Examines the effect of cord length on the stretch of the cord (\( k_c \)).

![Graph of \( k_c \) vs. \( \frac{1}{\text{cord length}} \)](image)

\( k_c = 1.42(1/l) + 0.27 \)

From the graph in Fig. 5, the following linear trendline can be determined:

\[
\text{Equation 6: } k_c = 1.42 \frac{1}{l} + 0.24
\]

Notably, we can see that the relationship is best described using a linear trendline. As cord length decreases (\( \frac{1}{l} \) increases), \( k_c \) increases. This is in accordance with our understanding of spring-like motion and the relationship described in Equation 4.

The standard error of the slope was found to be 0.07 and the uncertainty of the intercept to be 0.3.

**Discussion**

The purpose of this set of experiments is to gain an understanding of the behavior of a bungee cord. Our previous experiments have shown that the simple form of Hooke’s law (Equation 1) will not accurately model the behavior of the cord. Equation 3 will provide better approximations of how Force changes displacement for various lengths of cord.

This experiment served to provide more information on the static behavior the cord. Fig. 4 shows us that \( k \), or stretch of the cord, changes as a function of cord length. As the length of
the cord increases, it stretches more when acted on by the same amount of Force. In fact, the slope of Equation 4 can be though of as a value describing stretch per length of cord. In other words, each unit of cord length contributes to 1.12 units of length \( \left( \frac{1}{0.886} \right) \) in displacement. This value may be useful in extrapolation of the cord’s behavior at longer cord lengths that will be necessary for the Great Hall bungee jump, particularly because the relationship that it describes is quite linear.

In analyzing Equation 4, we chose to force the line through zero to simplify the model. We can justify this because when the cord has no length, it cannot possibly be displaced. It can also be justified because the uncertainty of the intercept, 0.02, is larger than the magnitude of the intercept itself, at 0.005. This means that the intercept is not significant. This assumption provides ease of analysis, but should also be used with caution as the y-intercept may account for actual cord behavior and not simply experimental error.

Operating under this same assumption can provide more insight into the behavior of the cord. Equation 6, calculated from the graph in Fig. 5, can be related to Equation 3 by again ignoring the y-intercept in Equation 4 and considering \( x = 0.886l \). We must also account for the exponent on \( x \), so the Equation 3 becomes:

\[
\text{Equation 7: } k_c = 1.421 \left( \frac{1}{(1.13x)^{3/4}} \right) + 0.241
\]

The total slope, 1.30, is found by combining numerical terms, represents the Force on the system (in N) applied by the mass. Based on our use of a 0.150 kg mass, the actual Force on the system is 1.47 N. The percent error for this relationship is 17%, a value too high to make this model experimentally useful. From linear regression, we know that percent uncertainty of slope is 5%, again demonstrating that the force calculated is not accurate.

Forcing the line in Equation 6 through the y-intercept produces the following equation:

\[
\text{Equation 8: } k_c = 1.512 \left( \frac{1}{(1.13x)^{3/4}} \right)
\]

Where experimental F becomes 1.40 N. We can justify setting y-intercept equal to zero because the uncertainty of the y-intercept is 0.3, which is larger than the y-intercept itself. This causes the percent error to drop to 7%. From linear regression, the percent uncertainty of the slope is 1%, meaning that this value is also not accurate.

From these sizeable percent errors, it’s evident that a great deal of experimental error exists. A small amount of this error could have come from the knotted loop regions of the cord. At these spots, the cord is double stranded instead of single stranded, so would have caused an increased resistance to stretch, or an increase \( k \). As the cord was continually knotted and re-knotted as length was changed, the differences in the different knots led to a lack of consistency.
Other error likely resulted from our rudimentary attempts at modeling a complicated system. For example, even the relationship expressed in Equation 3 is an experimental approximation of a power trendline with an exponent for of 0.691, not 0.75. This approximation was made in a previous experiment to simplify the Equation, but causes inherent error in future calculations using the model. Other such approximations, like setting the y-intercept equal to zero in Equation 4, could also contribute to error in later calculations. As we are creating a new model for a system, it is difficult to determine the effect of such approximations.

If we would like to use Hooke's Law to model bungee motion, we must improve our general model of the law for bungee motion (Equation 3), improve our relationship between x and l (Equation 4), or improve both. As the results currently stand, error is too high (as proved by comparison of calculated Force) to design a safe and accurate jump using our model. Although altering our model will likely increase complexity, it may be the only way to achieve satisfactory accuracy.

Finally, it is important to note that we chose to discard the data point at cord length 0.11 m in Fig. 5. This length had a $k_{\text{corrected}}$ much lower than fit the rest of the data. This may be due to poor measurement, as it was the first length measured. It may also be due to a change in the relationship between $k$ and displacement at lower cord lengths. However, this is less worrisome that a wayward point at high cord lengths, as the bungee jump will be taking place at lengths much longer than those in the lab.

**Conclusion**

This experiment provides information vital to the eventual survival of our egg. We found that a constant exists that provides stretch per length of cord at a particular mass. Importantly, we also found $k$ and $\frac{1}{\text{cord length}}$ to have a linear relationship. This too will prove useful if we decide to use a variation of Hooke's law when designing our final bungee jump.

As a final note, it is important to recognize that this experiment deals with a static system; the mass is not dropped from a height and observed, but rather is allowed to equilibrate and then displacement measured. However, the kinetic nature of the egg drop will need to be accounted for in our final design. This should be investigated in future pre-jump experiments.

Pledged in full.