Bungee Challenge Part 1: Bungee Cord Characteristics with Focus on the Elastic Behavior

**Introduction:** In order to better understand how to give our egg a safe landing when we do our final experiment we need to understand the materials that we are given and their traits. The experiment that we did focused on understanding the bungee cords and their functional relationship between the force and displacement. We then used this relationship, which is the spring constant of the cord, to decide if the cords’ behavior is linear enough to model it simply by using the model of a linear spring (Hooke’s law) to characterize the behavior of elastics.

Equation 1: Hooke’s Law \( F = -kx \)

Equation 2: Newton’s Second Law \( F = mg \)

The variable \( F \) in both equations is equal to the force acting on the cord which in this experiment was equal to the mass \( (m) \) added to the cord times the force of gravity \( (g=9.8m/s) \) \( (F=mg) \) which is proven in Newton’s second law which was the second equation. The variable \( k \) in the first equation stands for the spring constant of the bungee cord. The variable \( x \) in equation 1 is equal to the displacement of the end of the bungee cord from its equilibrium point.

**Methods:** Our overall method mainly consisted of us trying to find the spring constant of the bungee cord with different weights by using both equation 1 and 2. We found the spring constant by placing a weighted hook to one end of the bungee cord and the other to support arm and then using the force that we knew and the displacement of the end of the bungee cord.

**Set-up:** Our setup consisted of a lab table with a metal pole with a clamp at the top that stood at about 2.3 meters above the floor. We first tied a knot at about the middle of the bungee cord because it was too long to hang from either ends without shortening it. We then hung the knotted end of the bungee cord from the clamp atop the metal pole and tied another knot on one of the hanging ends of the bungee cord in order to have the ability to hang a weighted hook on the free hanging end. Next, we stretched the bungee cord out by pulling up and down on it a few times in order to make sure it didn’t get stretched out and have results that changed drastically during our

![Figure #1: Static Diagram of Bungee Cord. A diagram that shows the force acting on the bungee cord and the displacement from the equilibrium point (the red x) and its displaced distance (the black x). The equilibrium point is .93m in the first set of trials and .73m in the second set.](image-url)
later tests. We then measured the distance from the top knot to the knot hanging freely at the bottom to get the equilibrium position of the bungee cord, which was .93m in the first set of trials and .73m in the second set. These values were our un-stretched equilibrium positions.

**Procedure**- We began to add masses to the free hanging end and by tying a hook to the free hanging end and then adding masses onto the hook. Once the mass was added to the free hanging end of the cord we then made note of how far the weighted mass went down from the equilibrium position (.93m) in order to see how much displacement from the equilibrium position occurred with each force due to the added mass and gravity. Once the masses were added we quickly measured the displacement and removed the weights in order to prevent stretching and we then calculated both the force (using equation2) and the spring constant (finding k in equation1) by using the displacement that we found for each mass. We also did a second series of trials using the same setup as before except that for the second series of trials we used a different equilibrium point (.73m in the second set) in order to see if a trend in the first series of trials would appear again if we changed the equilibrium position.

**Results**

The main results from the experiment showed that there was a major decrease in the change in the k value for each mass once we reached about .1kg. This change in the slope that appeared around .1kg showed that a linear function for the graph as a whole wouldn’t work as well as another kind of function, possibly a polynomial function, because the linear function doesn’t fit the graph all that well and wouldn’t give results that are as accurate as a different kind of function due to the change in the slope for the function of k. This is apparent if you look at the data from figure#2.

**Figure#2:** Table of force, displacement, and spring constant values. These are the calculated values we got from our first set of trials at the equilibrium position of $x_o = .93m$

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Force (N)</th>
<th>x (length of stretch in meters)</th>
<th>Calculated &quot;k&quot; = F/X</th>
</tr>
</thead>
<tbody>
<tr>
<td>.01</td>
<td>0.098</td>
<td>0.03</td>
<td>3.27</td>
</tr>
<tr>
<td>.03</td>
<td>0.294</td>
<td>0.1</td>
<td>2.94</td>
</tr>
<tr>
<td>.05</td>
<td>0.49</td>
<td>0.18</td>
<td>2.72</td>
</tr>
<tr>
<td>.07</td>
<td>0.686</td>
<td>0.27</td>
<td>2.54</td>
</tr>
<tr>
<td>.1</td>
<td>0.98</td>
<td>0.46</td>
<td>2.13</td>
</tr>
</tbody>
</table>
During the first few trials (trials 1-4), as the force increased the spring constant (K) seemed to decrease rapidly but towards the later trials (trials 5-9) it appears that as the force increases the spring constant continues to decrease but begins to taper off and decrease at a decreasing rate or at a rate much slower than in the first 4 trials. This is really apparent when you look at the graph (Figure #3) of the force vs displacement from Figure #2.

Figure #3: Graph of force vs displacement. This graph shows the force vs displacement of the first set of trials and shows the linear lines for the points as a whole set and as the first set of 4 trials and the last set of 5 trials

As you can see if you try and fit a linear function (using our Equation 1) onto these results it really isn’t a good fit and we end up with a linear function of as

\[ F = 1.35(x) + 0.22 \]

with an uncertainty of ±0.57 N where F is the force on the system and x is the displacement from the equilibrium position and the slope in the equation is our spring constant (k). This can most
likely be accredited to the fact that the first 4 trials and the last 5 trials from the first set seem to form a stepwise function. By getting a linear fit for the first four as

\[ F = 2.63x \]

with an uncertainty of \( \pm 0.097N \) and last five trials as

\[ F = 1.06x + 0.5 \]

with an uncertainty of \( \pm 0.0186N \) we were able to clearly see and confirm that the data does in fact form a stepwise function. Once we noticed this abnormality in the data we decided to see if it would repeat itself if we changed the equilibrium point. We decided to use the new equilibrium point of \( x_0 = .73 \) and to do a few more trials with similar and greater weight to see if the trend continued and we recorded our results in a table (Figure #4.)

Figure #4: Table of Values for the Second Height for an Equilibrium Position. This is the set of data that we received when we changed the equilibrium position from \( x_0 = .93 \) meters to \( x_0 = .73 \) meters in order to test heavier masses.

<table>
<thead>
<tr>
<th>Mass (kg) +/- .01</th>
<th>Force (N) +/- .001</th>
<th>X (Length Displacement) +/- .001</th>
<th>K (F/X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>0.480</td>
<td>0.13</td>
<td>3.692</td>
</tr>
<tr>
<td>.07</td>
<td>0.686</td>
<td>0.19</td>
<td>3.610</td>
</tr>
<tr>
<td>.09</td>
<td>0.882</td>
<td>0.29</td>
<td>3.041</td>
</tr>
<tr>
<td>.11</td>
<td>1.078</td>
<td>0.39</td>
<td>2.764</td>
</tr>
<tr>
<td>.14</td>
<td>1.372</td>
<td>0.6</td>
<td>2.287</td>
</tr>
<tr>
<td>.16</td>
<td>1.568</td>
<td>0.75</td>
<td>2.091</td>
</tr>
<tr>
<td>.2</td>
<td>1.96</td>
<td>1.07</td>
<td>1.832</td>
</tr>
<tr>
<td>.22</td>
<td>2.156</td>
<td>1.21</td>
<td>1.782</td>
</tr>
<tr>
<td>.24</td>
<td>2.352</td>
<td>1.33</td>
<td>1.768</td>
</tr>
</tbody>
</table>

After looking at the new set of data and comparing it to the previous set using the other equilibrium point the trend did seem to continue. The table shows that for the first 4 trials the values for the spring constant did decrease rapidly compared to the final 5 trials which seemed to decrease at a decreasing rate. But by using a new equilibrium point that allowed for more weight we were able to notice that the change in the rates at which the spring constant decreased occurred around the mass of \( .1kg \pm .01 \). This becomes even more prominent when we look at the graph (Figure #5) of the second set of trials.
Figure #5: Graph of Force vs Displacement for the set of trials at the equilibrium position of $x_0 = 0.73\text{m}$ with a linear line fit for the force vs displacement for all the trials, for the first 4 trials, and the last 5 trials.

In the second set the linear function for the data came out to be

\begin{align*}
(F &= 1.46x + 0.43) \\
\text{with an uncertainty of } &\pm 0.66\text{N while the functions for the first 4 trials had a function of} \\
(F &= 2.2x + 0.23) \\
\text{with an uncertainty of } &\pm 0.017\text{N and the last 5 trials had a function of} \\
(y &= 1.317x + 0.575) \\
\text{with an uncertainty of } &\pm 0.036\text{N. Although the data of the second set isn’t as prominent as the} \\
\text{first set in showing that it is a stepwise function the data does show a similar shift in the} \\
\text{decreasing of the spring constant around the area of the graph where the mass becomes} \\
0.1\text{kg} &\pm 0.01\text{ or more. But by repeating the experiment with a different equilibrium position we did} \\
\text{see that this stepwise function is in fact a characteristic of the bungee cord.}
Discussion:

Now looking at all the data that we have amassed my partner and I decided that although we could use the linear function using all the information from the trials we hypothesized that using the linear function from the second half of the stepwise function could be much more accurate. Given the data from both groups of data we know that both are functional and plausible routes but each has their own merits. Although the linear function has a much larger uncertainty (with 36% for the first set and 34.9% for the second set) it does give a great general idea of what our bungee cord will do based on the force applied to it with a large array of masses. If we decide to use the two linear functions from the stepwise function we would have a much more precise and accurate calculation because of a much lower uncertainty for each function in the stepwise function (It was 3.69% for the function for the first 4 trials in the first set, 1.19% for the function for the 5 last entries on the first set, .69% for the function of the first 4 in the second set, and 1.9% on the function for the final 5). But in order to do that we need to know a variety of variables that we don’t know as of yet (the weight of the egg is currently estimated between 100-170g and the height of the jump is somewhere between 8-9 meters). But based on the data the second part of the stepwise function could be the better fit for the experiment because it was found using masses that ranged from 100-200g and it has a much smaller uncertainty, 1.9% for the function of the last 5 trials, which is much more reasonable than the 34.9% that we got when we used the linear function for the whole series of trials. This 1.9% uncertainty is the most reasonable out of the functions that we found in this lab to work with when we extrapolate to the actual egg drop because of its low percent uncertainty (1.9%), and the fact that the masses used to find it were within range of the eggs mass (100g-170g). Using this second linear function for K from the stepwise function of the second function rather than the linear function of all masses because we would have a better approximation of the spring constant for the cord and better know the elasticity of the cord for when our actual egg drop experiment occurs. Therefore, we believe that the function \(y = 1.317x + .575\) with an uncertainty of \(\pm .036N\) would be the best function for the k value to describe the cords elasticity.

Uncertainty could have been caused by the bungee cord being stretched out from carrying the heavier weights at the end of the first series of trials and would then be less elastic for the second series of trials. This probably explains the sharp slope for the first 4 values in the first series of trials that doesn’t seem to be as present in the second series of trials. This uncertainty could be eliminated if we hung our heaviest mass on our cord before our trials and stretched it out so that it wouldn’t cause a large variety in the results. Uncertainty could also occur due to the fact that we recorded our lengths by eyeballing the displacement measured on the ruler when measuring the hanging mass and would most likely cause our data for the displacements to be off by about a millimeter and would therefore slightly alter the data that went into forming the linear function.
functions. This uncertainty could be fixed by placing a much larger ruler behind the hanging cord and measure it that way rather than placing one next to it and eye balling it, or measuring the displacement multiple times and averaging the results we got.

**Conclusion:**

During our experiment we discovered the fact that the elasticity of our bungee cord was represented best by using a stepwise function. Therefore it can be a safe bet to state that if we wanted to use a model of a linear spring to characterize the elasticity of our cord we could try and use a linear model similar to the functions that we found in both our experiments. Since we noticed that the percent uncertainty was much lower for the stepwise functions it would be wise to use one of the stepwise functions. It would also be wise to use the functions form the stepwise functions that had data sets that covered the masses that ranged from .1kg-.17kg, because that will be the mass of our egg. These masses were mainly used in the 5th -9th trials of each data set. Using one of these linear functions for the spring constant we could then better understand the elasticity of our cord and better predict how far away from the equilibrium position the egg will go and how much force it will receive in its journey. Next we should focus on trying to see how the length of the cord affects the force-displacement relationship.