Bungee Lab 1

Introduction:

The purpose of this lab is to measure the relationship between the mass on an elastic cord, the length of the cord, and the distance traveled by the mass. Because the goal of the lab is to eventually drop a mass from a height attached to this cord and be able to determine the distance it will travel, it is important to figure out the “stretchiness” of the cord. We want a value for “stretchiness” to give us an equation where we could plug in a mass and length of this unstretched cord, and determine the distance the mass will travel.

Methods:

We used a mass on the elastic cord and measured distance traveled by the mass to figure out the relationship between the mass, the length of the cord, and the distance travelled.

Setup:

We used a hanger on the elastic cord, which we dropped from the attachment point of the cord. We recorded this on the iPad program CMV (Coach My Video) to slow down the video to see the max distance travelled by the hanger when released.

Procedure:
First, we stretched the cord out so that we could get the cord to a more constant un-stretched length, as cords have the property of hysteresis, a tendency of elastic cords to stay stretched out. We tied two loops in the cord, one at the bottom for the hanger, and one at the top to put on a hook on a stand. We attached the cord to the stand, and measured the length of the un-stretched cord from the top of the stand to the bottom of the loop ($L$, in m), which was $0.48 \pm 0.005$ m. We attached a $0.05 \text{kg} \pm 1\%$ hanger to the loop, and measured the length of the static displacement ($x_o$, in m). We then dropped the hanger from the attachment point of the elastic cord to measure the max dynamic displacement of the hanger ($x_{max}$, in m). We recorded the drop in an iPad using the program CMV (Coach My Video) to slow down the video so we could see the hanger move more clearly. It was easier to see the bottom of the hanger, so we measured to that point and subtracted the height of the hanger, which we measured to be $0.193 \pm 0.01$ m. We repeated using 7 masses varying from $0.05 \pm 0.0001 \text{kg}$ to $0.2 \pm 0.0001 \text{kg}$, and using 3 un-stretched cord lengths of $0.252 \pm 0.005 \text{m}$, $0.32 \pm 0.005 \text{m}$, and $0.48 \pm 0.005 \text{m}$.

**Results:**

We used the distance travelled by the mass to determine a value for “stretchiness” of this cord, which we found to be $16.9 \pm 0.6 \text{ kg}^{-1}$. This gives an equation for the relationship between the mass on this elastic cord, $m$ in kg, the length of the cord, $L$ in m, and the displacement of the mass, $x_{max}$ in m, which is

$$x_{max} = 16.9 \pm 0.2 \text{ kg}^{-1} \cdot L \cdot m$$

![Figure 1: Max Dynamic Displacement](image)

This table has the hanging mass and the unstretched length of the cord used in each trial, and the max dynamic displacement of the hanging mass when dropped.

<table>
<thead>
<tr>
<th>Mass (kg) ± 1%</th>
<th>$X_{max}$ (m) ± .005m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_1$ ($L=.252 \pm 0.005 \text{m}$)</td>
</tr>
<tr>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>0.07</td>
<td>0.28</td>
</tr>
<tr>
<td>0.1</td>
<td>0.405</td>
</tr>
<tr>
<td>0.12</td>
<td>0.505</td>
</tr>
<tr>
<td>0.15</td>
<td>0.62</td>
</tr>
<tr>
<td>0.17</td>
<td>0.71</td>
</tr>
<tr>
<td>0.2</td>
<td>0.835</td>
</tr>
</tbody>
</table>

We found the $x_{max}$ for each drop by measuring the max distance from the top of the stand to the bottom of the hanger, and subtracting the height of the hangar and the un-stretched length of the cord.

![Figure 2: Max Dynamic Displacement vs. Mass](image)

This graph shows the max dynamic displacement for each mass at each cord length, with trendlines and the equations for those three lines.
Knowing that the y-axis, which is the max dynamic displacement, is in m, and that the x-axis, which is mass, is in kg, that means the slope of the line is in mkg\(^{-1}\). So, the equations for the lines are

- \( L_1: x_{\text{max}} = 4.25 \text{ mkg}\(^{-1}\) * m \(-0.015 \text{ m} \)
- \( L_2: x_{\text{max}} = 5.60 \text{ mkg}\(^{-1}\) * m \(-0.03 \text{ m} \)
- \( L_3: x_{\text{max}} = 7.8 \text{ mkg}\(^{-1}\) * m \)

Using linear regression for the equations, we find that the standard error, which is the uncertainty, for the first equation for the slope is ±0.04 mkg\(^{-1}\) or 1% and for the y-intercept is ±0.005 m or 40%. The standard error for the second equation for the slope is ±0.09 mkg\(^{-1}\) or 2% and for the y-intercept is ±0.01 m or 40%. The standard error for the third equation for the slope is ±0.1 mkg\(^{-1}\) or 2% and for the y-intercept is ±0.02 m or 500%.

We define a value for “stretchiness”, \( Z \), for this cord as the ratio of the slope to cord length. If \( Z \) is the value of “stretchiness” for this cord (kg\(^{-1}\)), \( s \) is the slope (mkg\(^{-1}\)), and \( L \) is cord length (m), then

\[ Z = \frac{s}{L} \text{ and } s = Z \cdot L \]

So, for \( L_1, Z_1 = 16.9 \pm 0.04 \text{ kg}^{-1} \), \( Z_2 = 17.5 \pm 0.09 \text{ kg}^{-1} \) for \( L_2 \), and \( Z_3 = 16.2 \pm 0.1 \text{ kg}^{-1} \) for \( L_3 \), using least squares approximation for propagation of uncertainties. The meaning that the average \( Z \) is 16.9 ± 0.6 kg\(^{-1}\) or ±4%, using standard deviation for the uncertainty. This gives an equation for the relationship between the mass on this elastic cord, \( m \) in kg, the length of the cord, \( L \) in m, and the displacement of the mass, \( x_{\text{max}} \) in m, which is

\[ X_{\text{max}} = s \cdot m \]

\[ X_{\text{max}} = Z \cdot L \cdot m \]

\[ X_{\text{max}} = 16.9 \pm 0.6 \text{ kg}^{-1} \cdot L \cdot m \]
**Discussion:**

The equation we found is acceptable for this experiment. Our goal was to find an equation we could use to drop a mass knowing how far it will go, and we found an equation that has 4% uncertainty within the “stretchiness” value. Using the equation we found and plugging in the parameters of an egg drop, say a .150kg mass and a 9m height, we get that we should use this cord with the unstretched length of 2.5m ± 4% or ±.1m. While the uncertainty in that value is .1m, that is something we could account for in an egg drop by raising the cord to err on the side of caution based on our uncertainty, so this is an acceptable equation.

Looking at why we could ignore the y-intercepts in the equations, the expected value for the y-intercept for all of these would be zero, for no movement of the cord when there is zero mass. The values for the percent uncertainties for the y-intercepts of the equations, which are 40%, 40%, and 500%, and which puts the expected value of zero well within experimental uncertainty.

Sources of uncertainty were the variability in the movement of the hanger due to swaying, random movement, and shifting masses in the hanger, the difficulty of capturing the fast movement of the hanger on the video, the inability to completely start the hanger from rest or from the exact same place when releasing it by hand, and possible movement of the stand and tape measure when the hanger was in motion. To reduce uncertainty, it would help to use the electronic drop that we will be using during the egg drop.

**Conclusion:**

This experiment does find a value for “stretchiness” of the cord, making it possible for us to have predict distance travelled given a mass. To build on this, it would be useful to see the effect of using multiple lengths of cord, or tying knots in the cord.