Introduction:

The Bungee Lab I performed is meant to determine the relationships between a bungee cord of length L, its fully extended length h and k constant. These concepts are needed in order to give a full picture of a bungee jump in real life where a mass is attached to an elastic band (or bungee) and dropped from a significant height in order for the mass to experience the “thrill” of plummeting to the ground. The trick of a bungee jump is to get as close to the ground as possible without hitting. To do this the facilitators of the jump take into account the elasticity of the bungee and the mass attached to it in order to ensure the mass doesn’t hit the ground while obtaining maximum acceleration and maximum excitement. The facilitators use a variation of the Hooke’s Law Equation:

**Equation 1:**

\[ F = -k x \]

**Equation 2:**

\[ F = -mg \]

Thus, **Equation 3:**

\[ mg = k x \]

Where m is the mass of the object hanging at the end of the bungee, k is the restoration force of the bungee, x is the distance from equilibrium the mass has traveled, and g is the force of gravity. For this, we supplemented Newton’s Second law for F, the force so that we can calculate the k value when x is measured.
Methods:

This experiment required taking many measurements: taking the measurement of the bungee as it hung from the anchor point without the mass attached; the measurement of the equilibrium point when the mass was attached; and, using a filming app on an iPad, measuring how far the bungee stretched when the mass was dropped from the anchor point. The first two measurements only had to be taken once because they were unmoving and without fluctuation. The measurement of the maximum displacement, though, we measured twice and averaged to account for changing circumstances of the drop.

Set-up:

One end of the bungee cord is attached to the anchor point on an overhanging edge while the other end is connected to a mass. The mass is then dropped from the anchor point while an iPad records, allowing us to go back and watch a frame-by-frame video of the drop which provides us with our estimate of h.

Figure 1:

Bungee Drop of Mass Attached to Elastic String with a Human Holding an iPad Recording

the Drop: Pictured below is a visual representation for the setup of the experiment where a mass is dropped from the anchor arm while someone uses an iPad to record the drop and estimate h. With L being the height of the bungee without a weight on it, \(x_0\) is the distance the string is stretched past L when the mass is at equilibrium, \(x_{\text{max}}\) is the distance the bungee is stretched past L when the mass is dropped from the anchor arm, and h is L and \(x_{\text{max}}\) combined. This diagram also depicts Eq. 1 and Eq. 2 when they act upon the system.
**Procedure:**

We knotted one end of the bungee string into a loop and left that alone, giving us a constant point to hang from the anchor arm. We then tied a different knot in the string at varying lengths from which we could hang a mass and measure the values $x_0$ and $h$. From $h$, we will subtract $L$ to get $x_{\text{max}}$. Before placing the mass on the bungee, we measured how long the string was without a mass stretching it, $L$. Afterwards, we placed the mass on the end of the string not attached to the anchor arm and gently let it down to equilibrium to avoid unnecessary wear on the string. Once measured, we brought the mass up to the anchor point and let it drop; one person held and dropped the mass while the other recorded the drop with software on an iPad that allowed us to slow the recording down and watch frame-by-frame in order to determine how far the mass
dropped. We repeated this drop twice with each length L and took the average to gain an understanding of the behavior of the string when it stretches. We did this for seven different lengths of string, each time recording L, x₀, and h (after each average h, we subtracted L from h to get x_max). Once all 14 trials concluded, we plotted the data in Excel.

Results

This experiment consisted of two major parts that will help us determine the length of bungee L required for a drop of certain height h and the x_max of that bungee. Similarly, we were able to extrapolate an equation that shows the relationship between the spring constant k and the x_max of the bungee, giving us an idea of how the bungee reacts to differing lengths. These factors allow us a picture of the motion of the system.

Figure 2: Effect of a 0.050 ± 0.0001 kg Mass on a Bungee at Different Lengths L

<table>
<thead>
<tr>
<th>L (m± 0.001)</th>
<th>h (m± 0.001)</th>
<th>x₀ ± (m± 0.001)</th>
<th>x_max (m± 0.001)</th>
<th>k (N/m) @ x_max</th>
</tr>
</thead>
<tbody>
<tr>
<td>.122</td>
<td>.194</td>
<td>.011</td>
<td>.072</td>
<td>.0353</td>
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<tr>
<td>.858</td>
<td>1.60</td>
<td>.115</td>
<td>.745</td>
<td>.365</td>
</tr>
</tbody>
</table>

In the first column, depicted in ascending order are the lengths of the bungee without a mass on it (L) in meters for the seven different trials, measured before the drops began on a flat table with a measuring tape accurate to the 0.001m, giving us the uncertainty for each measurement made.
with it. \( h \) was determined using the iPad app that recorded the drop and allowed for a frame-by-frame recap of the drop; using this technology we were able to estimate the maximum of the drop. For this value, we took the measurement twice and averaged it to get a better idea of where the true maximum was. \( X_0 \) was found when we placed the mass on the end of the bungee and allowed it to rest in equilibrium with no motion in the j-hat direction while we measured the bungee’s new length. \( X_{\text{max}} \) was found using the relationship \( h = L + x_{\text{max}} \). The spring constant, \( k \) was calculated using eq. 3, where we knew the mass of the hanger was \( 0.050 \pm 0.0001 \text{ kg} \) and gravity was \( 9.81 \text{ m/s}^2 \) – we used the position \( x_{\text{max}} \) for the \( x \) portion of the calculation.

**Graph 1: Length of Bungee \( L \) and its Relationship to the Height of the Fully Extended Cord \( h \)**

![Graph 1](Image)

Graph 1 plots the points for \( L \) against their corresponding \( h \) values. The relationship between them is positive (as \( L \) increases, so does \( h \)) which is expected. The linear regression equation for this graph, in context is \( h = 1.87L - 0.0382 \). In verbal form, this means whenever \( L \) is multiplied by the scalar 1.87 and that quantity subtracts 0.0382 meters, the result is equal to the total length.
of the bungee from its anchor point to its maximum stretch of h meters. Thus allowing us to predict how far the mass of 0.0500 kg will fall for any given length of L. Standard Error of length L is approximately 0.0354m, meaning that this equation is accurate within 0.0354m of the calculated value for h in either direction.

**Graph 2: Length of Bungee L and its Relationship to the Spring Constant k at x_{max}**

Graph 2, although appearing to be the same line as either Graph 1, is fundamentally different, as there is a separate equation relating the values of original length and spring constant than there is relating L and h or x_{max}. The equation for this graph is $k = 0.428L - 0.0187$, where k is the changing elasticity of the string as it grows in length; that is to say that the longer L gets, the farther it stretches. This equation is needed to determine the amount of restoration force the bungee has at any length L. The slope of this equation is the portion of L that k is at the distance x_{max}. Standard Error of the L value for this equation is approximately 0.173m.

**Discussion**
The equations derived from Graphs 1 and 2 serve to create a mathematical model of the experiment and allow us to continue with similar experiments. Although in this experiment, we wanted to find the relationship between \( L \) and \( h \) or \( k \), which we did. It was learned that the relationship between the two sets of variables is linear and even gives us Standard error from which we can determine how far off a calculation can be from reality without challenging the integrity of the experiment.

The formula derived from Graph 1 allows us to calculate the total extension \( h \) of a bungee of length \( L \) for a mass of \( m = 0.050 \) kg. That is why we needed the formula from Graph 2 as well, to make way in our calculations for a change in mass. This poses a problem, because as mass changes, so will the force applied to the bungee, thus there will be a different resulting \( x \) for every length \( L \). Similarly, because \( k \) is not constant (changes with every different displacement \( x \) because it takes different amounts of force to restore the mass to equilibrium) we cannot calculate the effect of a different mass on our bungee, revealing the shortcomings of our experiment. Yet, this is not the end. If we found the new mass as a proportion of the old one, we could theoretically add bungees with similar properties to account for the change, rendering the difference in mass null. (Say we had a new mass that was \( 3x \) the weight of the old one, we could just add \( 3x \) the amount of bungee cords of the same properties to return to functionality our equations.)

In this experiment, as with all, there is uncertainty. Perhaps the most prominent source in the Bungee Lab I was the fact that the bungee cord never fully restores to the same exact state. That
is, over time the more the bungee is stretched, the more it will remain stretched, not bouncing back to its original elasticity. This process is called hysteresis. Thus, the more the bungee is used for drops or experiments, the more the components of the bungee changes, namely the restoration force. Other sources of uncertainty could be an unclean fall, meaning that the system is not aerodynamic at some point in the drop, allowing friction to slow the mass until terminal velocity is reached, in effect giving less a thrill. Or perhaps the mass was not dropped from the same point each time, creating a drop time discrepancy and allowing for differences in acceleration.

To minimize these errors, one could attempt to place no more strain on the bungee than necessary. Likewise, one could use a mechanism that cradles the mass until drop, keeping it at the same height and alignment each time. If this experiment could be redone, I would use different masses with the same length L and measure the changes in $x_0$, k and h to get a better idea of how the bungee reacts to changing masses to create a better system with which to determine how long L must be for a drop h with mass m.

**Conclusion**

Although not enough information was gathered to create a model for every potential drop with this bungee cord, we have gained significant insight on how to improve the experiment and gather the missing information. With what we have, we can predict h and k from any length L with a mass of 0.050kg, having a functional model and set of equations that can be utilized for this purpose.