The Static Relationship Between k and L: More than Neighbors in the Alphabet
Section 01
November 5, 2015

Introduction

This experiment is the first one part of a series of experiments in which the ultimate goal will be to decelerate a falling egg using only spring force. Because spring force is dependent on stretch distance and spring constant by Hooke's law (equation 1), we wanted to understand the relationship between the length of the unstretched elastic string and its spring constant so that we will be able to manipulate the spring constant by changing the length of our elastic. By varying the initial length of the string and applying a constant force, we were able to calculate the spring constant using the amount the spring stretched. To see this relationship, we kept mass constant. We also only examined the static spring constant. We used the equation

\[ F_{spring} = -k \times x \]  

(1)

where k is the spring constant and x is the distance from the equilibrium, but in this case we focused on variability in k depending on resting length. In this experiment, we will measure the stretched and unstretched lengths of an elastic string and after calculating k using Hooke's law compare unstretched length against the spring constant.

Methods

In this experiment, we changed the length of the elastic string and measured the stretch to find the spring constant, Hooke's law. Our setup involved hanging an elastic string from a tall overhang and measuring the change in stretch for different lengths of starting elastic using a constant mass of .15 kg, similar to the mass of an egg, to stretch the elastic string using the gravitational force.

First, we manually stretched out the elastic string by pulling it taut. This minimized error created by helping the elastic string to behave more consistently. Then, we tied it with a small, tight knot to the bar from which we hung it. To vary the length of the string while experimenting, we changed the length of the elastic string by tying a knot in the elastic with a small loop that stretched a negligible amount. We measured the section between the top knot connecting the elastic string to the bar from which it was hanging to the bottom knot, tied close (less than 10 cm) to the top knot. Since our data is graphed, having perfectly even intervals between data points is not important as long as they are well spread out over the length of elastic string being tested. We chose "well spread out" to be about .05 to .10 meters between test lengths.

First, we measured the stretch in the string from the base of the knot attached to the hanger to the base of the knot that was not yet holding the mass in order to get initial length. We did not pull the string or stretch it significantly. We were sure to elevate the tail of the elastic string so it did not affect our results by applying a slight force to the top part of the elastic string. Then, we attached .15 kg of mass to the base of the string and measured the new static stretch to find the change in the equilibrium length. After recording equilibrium length with and without the mass, we untied the knot by slowly pulling on both loose strands creating the knot and retied the knot between 4 and 10 centimeters further from the original knot and repeated the procedure, recording our results in Excel. The only data we recorded was length of elastic string with no
attached mass and length of the elastic string with attached mass; using these variables and the added mass, we calculated everything else from these numbers.

![Diagram of experiment setup.](image)

Figure 1: set up of experiment. Circles represent knots, square represents a .15 kg mass. The left is the initial, massless length, and the right is the new equilibrium with mass. Note how the knot was tied before the mass was added. The wavy line represents the elevated tail of the elastic spring, kept out of the way so as not to influence results.

Results

In this experiment we discovered that the relationship between spring constant and unstretched elastic string length is inversely proportional by the following relationship:

\[
    k = \frac{1.6858}{L}
\]  

where \( k \) is the string constant and \( L \) is the unstretched length. To arrive at this equation, we first calculated the gravitational force on the system by assuming that the mass of the string was negligible. We multiplied the .15 kg mass by \( g \) and used this force and the distance stretched to calculate the spring constant using Hooke's law (equation 1).
<table>
<thead>
<tr>
<th>Length with no mass (m) (L) ±.0001</th>
<th>Length with .15 kg (m) ±.0001</th>
<th>change in length (m) ±.0002</th>
<th>k (Force, 1.47 N, over change in length) ±.0002</th>
<th>1/L ±.0001</th>
<th>F=1.47 N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.089</td>
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<td>0.078</td>
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<td>11.23595506</td>
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<td>0.765</td>
<td>1.921568627</td>
<td>1.11111111</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: data collected from experiment

In figure 2 we have the data collected from our experiment. The first column is the string length without mass. The second column is the string length with mass, and the third column is how much the string stretched from the added mass, or the difference of the second column minus the first column. The fourth column is a calculation of \( k \) found using Hooke's law (equation 1) and the fifth is \( 1/L \) divided by column one, which was useful in graphing the data and finding a relationship in figure 4. Uncertainty in the first two length columns come from the precision uncertainty inherent in our measuring tape. The uncertainty in the third column is twice as large because there is uncertainty about both the true unstretched length of the elastic string and the true stretched length of the elastic string. Because change in length is multiplied by constants to arrive at spring constant, \( k \) has the same uncertainty as the change in length, and because \( 1/L \) uses the same measurements as \( L \) it also has the same uncertainty.
Next, we graphed $k$, the spring constant, vs $L$, the unstretched length (figure 3). The exponent of the power equation, $-0.984$, is quite close to negative one. We came to the conclusion that $k$ and $L$ are inversely proportional.

$$k = 1.7365(L)^{-0.984}$$

**Figure 3**: graph of spring constant vs. unstretched length (power function)

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Next, we graphed $k$, the spring constant, vs $1/L$, the unstretched length (figure 4). The exponent of the power equation, $-0.984$, is quite close to negative one. We came to the conclusion that $k$ and $L$ are inversely proportional.

$$k = 1.6858/L$$

**Figure 4**: graph of spring constant vs. $1/$unstretched length (linear function)
To test this conclusion, we graphed \( k \) vs. \( 1/L \) and got a function that is almost perfectly linear through the origin (figure 4). We are able to say that for a mass of .15 kg on this type of elastic string, the relationship between \( k \) and \((1/unstretched\ length)\) can be described using a linear function.

Discussion

Our results surprised us because we showed that length and spring constant were inversely proportional. Until now, we had never looked at the relationship between length and spring constant before so we had not formed a hypothesis. The slope uncertainty of figure 4, the final linear graph proving the inverse proportion between spring constant and unstretched length, was .0066, or .39%. The slope uncertainty of our slightly more precise power model, figure 3, was .0046, or .26%. In addition to this, our measuring tape only went to tenths of a centimeter, so our measurements have additional uncertainty of ±.0001 or ±.0002.

We had very low percent errors, and because the procedure involved only the relatively simple task of measuring static objects we expected our measurements of error to be small. By tying very tight knots, we also minimized error by having almost zero excess stretch. Another step we took to minimize error was deciding to omit a piece of data that while maintaining the general trend was not representative of our overall findings. We attributed the cause of it to be poor measuring and took it off our graphs when calculating spring constant which dramatically increased the precision of our data without changing the trend. More sources of error in our experiment could have included our decision to exclude the lengths of the knots from our measurements of elastic length or small variations in the elastic we were testing by tying knots too tightly or stretching it too far.

Conclusion

Our experiment proved that there is a nearly perfect inverse relationship between spring constant and length of spring for this particular elastic. This will be useful in dropping our egg because we know now that a long stretch of elastic could decelerate the egg too slowly and allow it to crack on the floor while too short a stretch might cause the egg to slow down at too quick a rate and crack the egg. In the future, we could expand on this data by keeping the length constant and learning about the relationship between distance fallen and amount of stretch when the mass is dropped from different heights to better understand the mechanisms of dynamic elasticity. It would be useful to know if static and dynamic spring constants are related or independent.