Bungee Challenge: Cord Characteristics

Introduction

Hooke’s law states that a constant factor of a spring, the resistance to move, affects the distance by which a force compresses or extends the spring. The constant factor of resistance is $k$ and can be seen in the equation:

$$\vec{F}_{spring} = -kx \quad [1]$$

The force is negative because it is going in the negative direction in relation to the system, and the spring is being extended. If the spring is being compressed, it would be in the positive direction. To find the value of $k$, the $\vec{F}_{spring}$ needs to be broken down. Using Equation 1, the purpose of this experiment is to test different masses on a length of bungee cord, and measuring the displacement. The experiment encounters a mass, weight, normal force, and the constant factor $k$, and leads to the equation:

$$-mg = -kx \quad [2]$$

Testing different masses and recording the resulting displacement $x$ will lead to the constant factor of resistance, $k$. The goal of this experiment is to find that $k$ value in relation with the displacement.

Furthermore, doubling the bungee cord will change the factor $k$, so testing different lengths of doubled cord will change the $k$ value, and provide more information on the characteristics of the cord. The goal of the second experiment is to determine what doubling the length will do to the $k$ value.
Methods

The first part of the experiment was carried out by applying different masses to a specific length of mass, and measuring the displacement of the length of the bungee cord. The second part of the experiment consisted of doubling up the cord at certain lengths, and keeping the mass constant to see how the value $k$ changed.

Figure 1: Diagram of Experimental Set Up of Bungee Cord and Hanging Mass

For the first part of the experiment, the bungee cord was hung on an extended arm of a stand, and different masses were placed at the end of the cord as seen in Figure 1. The experimental lengths of the cord were 0.475 m, 0.69 m, 0.932 m, 0.198 m, 0.572 m. A tape measure was used to measure the different lengths of cord. A set of masses were used to test a length, and this set of masses remained the same with each length. They were 0.10 kg, 0.11 kg, 0.12 kg, 0.13 kg, 0.14 kg, 0.15 kg, 0.16 kg, and 0.17 kg. A length of cord that was hanging would be measured out to one of the selected lengths without mass on it. Then, one at a time, the masses would be hung on the length of cord, and the resulting length would be measured. The
displacement between the original length and length with mass was the main recorded value in meters. Every single mass was tested with every length. Then the data was graphed, and a linear trend line, slope, and R² valued was found. Using the mass (m) and displaced length (x), the k value can be calculated from Equation 2.

The second experiment was set up similar, except it tested different lengths of cord that were doubled up. There was a length of cord that remained the same that a mass (0.15 kg) was attached too, and this length was 0.582 m. Above this length was where another length of cord existed, and this was where the cord was doubled up at different lengths. The double cord lengths were 0.208 m, 0.255 m, 0.35 m, 0.45 m. The mass remained the same for each length. The displacement between the constant length (0.582 m) and the length after the mass was attached was the main value recorded. From these data points, a graph was made where a linear trend line, slope, and R² value was found.

**Results**

<table>
<thead>
<tr>
<th>Normal Length (m)</th>
<th>Mass (kg)</th>
<th>Displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.198</td>
<td>0.100</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>0.110</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>0.120</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>0.130</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td>0.140</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 1: Raw Values From Experiment 1
<table>
<thead>
<tr>
<th></th>
<th>0.110</th>
<th>0.201</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.120</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>0.130</td>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>0.140</td>
<td>0.322</td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td>0.360</td>
<td></td>
</tr>
<tr>
<td>0.160</td>
<td>0.402</td>
<td></td>
</tr>
<tr>
<td>0.170</td>
<td>0.445</td>
<td></td>
</tr>
<tr>
<td>0.69</td>
<td>0.100</td>
<td>0.272</td>
</tr>
<tr>
<td>0.110</td>
<td>0.311</td>
<td></td>
</tr>
<tr>
<td>0.120</td>
<td>0.367</td>
<td></td>
</tr>
<tr>
<td>0.130</td>
<td>0.422</td>
<td></td>
</tr>
<tr>
<td>0.932</td>
<td>0.140</td>
<td>0.4072</td>
</tr>
<tr>
<td>0.150</td>
<td>0.366</td>
<td>0.43</td>
</tr>
<tr>
<td>0.160</td>
<td>0.286</td>
<td>0.497</td>
</tr>
<tr>
<td>0.170</td>
<td>0.3350</td>
<td>0.565</td>
</tr>
<tr>
<td>0.140</td>
<td>0.633</td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>0.160</td>
<td>0.793</td>
<td></td>
</tr>
</tbody>
</table>
Table 1 shows the values of displacement from the normal length for each mass tested at each length. With increasing normal length and mass, there is more displacement.

<table>
<thead>
<tr>
<th>Normal Length (m)</th>
<th>R²</th>
<th>Slope (m/kg)</th>
<th>Inverse Slope (kg/m)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.198</td>
<td>0.992</td>
<td>1.5</td>
<td>0.666666667</td>
<td>6.54</td>
</tr>
</tbody>
</table>

Table 2: Calculated Values To Find $k$

<table>
<thead>
<tr>
<th>Normal Length (m)</th>
<th>$R^2$</th>
<th>Slope (m/kg)</th>
<th>Inverse Slope (kg/m)</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.170</td>
<td>0.878</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Values from Table 1 were plugged into slope and $R^2$ functions. The inverse slope was taken to get the units into (kg/m), and then the inverse slope was multiplied by the magnitude of gravity to obtain $k$.

$$y = 12.865x^2 - 21.423x + 10.22$$

Figure 2 shows normal length verses its $k$ value. There is a decrease in $k$ with the increase in length of the cord. The graph shows a negative correlation between the two variables.
Table 3: Raw Data for Experiment 2

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Normal Length (m)</th>
<th>Varying length (m)</th>
<th>Displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.582</td>
<td>0.45</td>
<td>0.234</td>
</tr>
<tr>
<td>0.15</td>
<td>0.582</td>
<td>0.35</td>
<td>0.298</td>
</tr>
<tr>
<td>0.15</td>
<td>0.582</td>
<td>0.255</td>
<td>0.333</td>
</tr>
<tr>
<td>0.15</td>
<td>0.582</td>
<td>0.208</td>
<td>0.349</td>
</tr>
</tbody>
</table>

Table 3 shows the raw values obtained from Experiment 2. The mass and normal length remained constant. With a decrease in parallel length, there is an increase in displacement.

Figure 3: Varying Parallel Length vs. Displacement

\[ y = -0.47x + 0.4519 \]
\[ R^2 = 0.9768 \]
Figure 3 shows varying parallel length against the displacement that resulted after the addition of a mass (0.15 kg). There is a decrease in displacement with an increase in parallel length. There is a negative correlation between the two variables.

**Discussion**

By varying the lengths and testing the set of masses at each length, the value $k$ was found in Experiment 1. There is a different $k$ value associated with each normal length that was measured. The data resulted in what would be expect. The more mass you place on the cord, the more weight it has, and the more displacement it will cause (Table 1). Also, the smaller the normal length is, the larger the $k$ will be. Meaning, there is more resistance against the displacement for a small amount of cord, and less resistance for large lengths of cord (Figure 2).

Experiment 2 represents what happens if sections of the cord are doubled up or parallel. When the parallel section is longer, it causes less displacement, which means the $k$ value is combining to cause more resistance to move.

The error may have occurred in the measuring of the lengths; however, measuring was done in centimeters so the uncertainty is far too small to make a difference. Also, in Experiment 2, only the length of the normal length was recorded after the mass was put on, meaning there was the length of the parallel that was not measured, and probably also had a displacement.
Conclusion

Experiment 1 proves the value of $k$ can be found. This is useful, because now, if a length was given, the $k$ and $x$ values could be found based in the equation calculated from Figure 1. Experiment 2 is useful because it means the more resistance can be caused by doubling the cord up. Lastly, this is all relevant for objects that going bungee jumping. Hooke’s law can be used, to an extent, to calculate the displacement of the cord at a certain length, so that the object do not crash into the ground.