Determining Variables of Consideration in the Development of an Elastic Bungee System

Introduction:

A relationship exists between the displacement of a spring’s displacement (x) and the amount of force applied to stretching it (F) – this directly proportional relationship is defined by Hooke’s Law:

\[ F = -kx \] (1)

We know this relationship to be true for springs, but it is uncertain how valid this approximation would be for an elastic cord. The first purpose of this experiment is to determine the relationship between the force applied on our bungee system and its displacement. Additionally, we are unsure how altering the length of stretched spring might affect the Force-displacement relationship, and so the secondary purpose of this experiment is to determine a relationship between the cord’s length and displacement. Thus, we are testing two unique variables (force, then length) in order to determine how we may consider them in proceeding with the development of our bungee system.

Methods:

Materials Needed: Set-Up:

Lab Bench with a Fixed Arm (from which to tie the cord)
Bungee (Elastic) Cord
Hanging Mass with Hook
Varied known masses
Meterstick/Tape Measure

Figure 1: Experimental Set-Up of Part I – Bungee with a single knot of known displacement and a variable hanging mass

Figure 2: Experimental Set-Up of Part II – Bungee with multiple knots of variable displacement and a known hanging mass
Description of Set-Up:

This rig was set-up in a typical laboratory/classroom setting at a lab bench. The bungee was attached to the fixed arm and five knots were tied at varying positions along the cord. Attention was paid to ensuring the knots would be able to hold the hanging mass by ensuring there was a space to “hook” the mass into the knot. We used a tape measure and hung it from the top of the arm to have a consistent measure of vertical displacement in the knot.

Procedure:

Part I:
To begin this experiment we set up our fixed system by tying five knots at varied locations along the elastic cord and then attaching the top of the cord to the rig so that the cord would hang vertically downwards. We then measured the distance of the middle knot from the location at the top of the cord where it ties to the rig (the origin point of the cord’s stretching). Then, we attached our hanging mass to the cord by looping the mass’s hook into the middle knot and recorded the final displacement of the cord in the vertical direction. We then repeated this measurement for varying masses by adding mass to the hanging mass, and recorded the displacement for each, also then calculating the force applied on each hanging mass by gravity (Figure 3).

Part II:
In this part, the focus of the experiment shifts towards determining a relationship between the length of the elastic and its displacement. To begin, we measured the displacement of the top knot on the elastic cord (the knot’s initial length value) from the origin point of the cord’s stretching, at the top of the rig. Then, we hooked the hanging mass (At a fixed 70 g) into the knot and measured its new length. We then subtracted the final length from the initial length to determine displacement, and then repeated this procedure for each knot, recording the results of each trial (Figure 4).

Results:

<table>
<thead>
<tr>
<th>$Y_f$ (cm) +/-1.0 cm</th>
<th>$m$ (g) [exact value]</th>
<th>$\Delta y$ (cm) +/-1.0 cm</th>
<th>$F$ (N) [calculated]</th>
</tr>
</thead>
<tbody>
<tr>
<td>67.0</td>
<td>50</td>
<td>13.5</td>
<td>.490</td>
</tr>
<tr>
<td>75.5</td>
<td>70</td>
<td>22.0</td>
<td>.686</td>
</tr>
<tr>
<td>91.5</td>
<td>100</td>
<td>38.0</td>
<td>.98</td>
</tr>
<tr>
<td>104.0</td>
<td>120</td>
<td>50.5</td>
<td>1.18</td>
</tr>
<tr>
<td>125.0</td>
<td>150</td>
<td>71.5</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Figure 3: Resultant data of experiment to compare a varying force with the cord’s displacement. Each trial was performed by hanging a mass at a knot displaced 53.5 cm (+/- 1.0 cm) from the top of the cord.
In each trial we varied the hanging mass and measured the final location of the knot on the elastic cord – the measurement for displacement was achieved by subtracting the final position of the knot from its initial position (53.5 cm). The force of gravity on the hanging mass was also reported for each trial, which is equivalent to the \( m \) (in kg) multiplied by the value \( g \), which is the value of the acceleration due to Earth’s gravitational field at surface level (=9.8 m/s\(^2\)). Theoretically, this force should directly relate to the measured displacement of each trial.

The measured values of displacement (Y-values) against the respective calculated values of \( F_g \) (X-values) that are acting on the hanging mass, and allowed Excel to create a power-model regression line for the values. (An additional point was added at a negligibly near-zero point in an attempt to produce a more accurate model – It is evident that when zero force is applied to the hanging mass, the displacement of the cord is also 0)

![Force vs. Displacement](image)

Figure 3: Graphical Comparison of the Knot’s Displacement and the Force of Gravity Acting on the Hanging Mass. Using a power model curve provides an incredibly accurate representation of the trend that relates these two variables.

Plotting the force acting on the hanging mass against the displacement of the knot graphically allows us to determine a relationship between them. We plotted the measured values of displacement (Y-values) against the respective calculated values of \( F_g \) (X-values) that are acting on the hanging mass, and allowed Excel to create a power-model regression line for the values. (An additional point was added at a negligibly near-zero point in an attempt to produce a more accurate model – It is evident that when zero force is applied to the hanging mass, the displacement of the cord is also 0)

<table>
<thead>
<tr>
<th>( L_i ) (cm) +/- 1.0 cm</th>
<th>( L_f ) (cm) +/- 1.0 cm</th>
<th>( \Delta L ) (cm) +/- 1.41 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.0</td>
<td>20.0</td>
<td>6.0</td>
</tr>
<tr>
<td>29.5</td>
<td>41.8</td>
<td>12.3</td>
</tr>
<tr>
<td>52.5</td>
<td>73.5</td>
<td>21.0</td>
</tr>
<tr>
<td>72.3</td>
<td>101.5</td>
<td>29.2</td>
</tr>
<tr>
<td>104.0</td>
<td>143.5</td>
<td>39.5</td>
</tr>
</tbody>
</table>

Figure 4: Resultant data of experiment to compare the elastic cord’s initial length and displacement for a given hanging mass. Each trial was performed by hanging a 70 g mass on a knot initially displaced \( L_i \) from the top of the elastic.
Each Lₐ represents a knot of a different initial length (Lᵢ) from the top of the system. In each trial, we hung the hanging mass of 70 g on each of the knots individually and then measured the final distance of the knot from the top. We then subtracted each final length (Lₐ) from each initial length in order to determine the extent to which the elastic cord was displaced in each trial.

![Length vs. Displacement graph]

**Figure 5:** Graphical Comparison of the Knot's Displacement and the Force of Gravity Acting on the Hanging Mass. Using a power model curve provides an incredibly accurate representation of the trend that relates these two variables.

Plotting the initial length of the elastic being stretched against the displacement of the knot graphically allows us to determine a relationship between them. We plotted the measured values of displacement (Y-values) against the respective measured values of Lᵢ (X-values) for the hanging mass at various knot locations, and allowed Excel to create a linear regression line for the values.

**Discussion:**

It is immediately obvious that there exists a strong correlation between the variables in both of the graphical representations of the data sets. While a power model represents one and a linear model represents the other, both R² values are nearly 1 exactly, and so the best-fit equations calculated by Excel can be regarded as reliable (Figure 3, 5). While some sources of uncertainty existed in our measurement, the degree to which it impacted our overall calculations was minimal, as only the measured length had a raw, and relatively low, uncertainty – this is further supported by the R² values achieved by the models, which were both, again, almost exactly equal to 1. Hooke’s Law, therefore, might not provide the most accurate model for our bungee system, because the Power model (rather than a linear one) had an R² value that =1 almost exactly. If Hooke’s Law should be assumed however, what can be deduced about it from Part II is that k varies with the length of the cord, as there exists a definite change in the displacement value in relation to the length of the cord being stretched, something that is not accounted for in Hooke’s Law (Formula 1). However, the compounding nature of these two variables makes it difficult to accurately determine the displacement of a cord just by knowing its length and the force expected to act upon it. What would be necessary would be a few data points at either the
expected cord length or while the cord is being stretched by the expected force. The information we can draw from these experiments is the qualitative knowledge of the existence of relationships that relate these three variables.

Conclusion:

It is evident from the results of this experiment that there exists a definite relationship between the displacement of an elastic cord and both the force acting on it as well as the length of cord being stretched. From Part I, we obtained an experimental model nearly exact in its precision to represent the relationship between the force acting on the knot and its displacement. In Part II, we obtained a similarly reliable model to represent the relationship between the initial length of the elastic cord and the amount it stretches additionally. To further visualize these relationships, it would be beneficial to carry out more procedures in the style of Part II but with several masses, in order to develop a three-dimensional model of the relationship among all three quantities; length of the elastic cord, the force acting upon it, and its displacement. This three-dimensional model might then allow us to accurately predict the displacement of the elastic cord given only its expected length and the force expected to act upon it. In the context of our bungee project, these qualitative results will become invaluable to us as we start to develop our bungee system, as we now know how length of the elastic cord and weight of the egg must be considered when theoretically determining how to achieve the maximum displacement without hitting the floor.

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-Michael Ian Colavita