Determining the Spring Constant, Modeling a Bungee Apparatus, and Preliminary Testing

I. INTRODUCTION

Hooke’s law governs the motion of springs and relates force \( F \), as exerted by a mass suspended from the spring, the spring constant \( k \), and the displacement of the spring \( x \), as a result of the application of the force.

\[
F_{\text{spring}} = -kx
\]  

(1)

Previous experiments determined that the bungee provided for the egg drop experiment does not follow Hooke’s law; the spring constant changes relative to the mass suspended by the bungee cord. This experiment aimed to mathematically model how varying the length of the bungee, rather than the suspended mass, causes variation in the spring constant. This model will allow extrapolation of the spring constant for any length of bungee cord. From this data, a final set-up for the bungee jump was determined and this model tested.

II. METHODS

To determine a function to describe the spring constant at various lengths, the set-up illustrated in Figure 1 was created in the lab. A loop was created in a length of bungee to attach it to a horizontal hanging structure. A similar loop was tied at the bottom of the bungee. A 50.0g hanger, to which additional mass was added, was suspended from this loop. The horizontal hanging structure was connected to a long metal pole that was, in turn, attached to the bench top. Four different lengths of bungee were used. The displacement resulting from the addition of six different masses was measured for each length of bungee (Table 1). Displacement was calculated by subtracting measurements taken of the bungee’s length with and without additional mass. A spring constant was determined for each length of bungee from the slope of the linear trend line (Figures 2-5). These slopes were then graphed against their corresponding lengths and fit with a power fit trend line (Figure 6). This produced the following equation relating the length of the bungee cord \( Y \) and the spring constant \( k \):

\[
y = \frac{0.762}{k}
\]  

(2)

From this equation, the spring constant of the bungee can be extrapolated for any length of bungee chord.

IV. RESULTS

The first part of this experiment successfully produced the spring constant values listed in Table 1. All trend lines created in this portion of the experiment had \( R^2 \) values greater...
than 0.98, indicating a high level of precision in the data set. Additionally, their uncertainty values, as determined by regression analysis, fell below ± 0.05. This small uncertainty further indicates the precision of data set. Further manipulation of this data, as explained in the methods section above, resulted in Equation 2. The power fit trend line that generated this equation had an $R^2$ value greater than 0.99, indicating that this equation is, indeed, a good model for predicting the spring constant at any given length. Uncertainty for this function, as determined by regression analysis, was ±0.1.

V. DISCUSSION

Results of the first part of this experiment indicate a downward trend in spring constants as the length of the bungee increases. This relationship is modeled with greater precision by Equation 2. This relationship makes intuitive sense. Much like a small rubber band is harder to stretch than a large rubber band, a smaller piece of bungee is more difficult to stretch than a long segment of bungee. This ease of stretching corresponds to a decrease in spring constant.

The data gathered above can be applied in the development of a safe bungee cord set up to be used in the final egg drop. The set up for the final drop will be similar to the experimental design of the experiment above. As illustrated in Figure 7, the egg will be placed in a harness and suspended from a segment of bungee. This bungee will be attached to a length of static line that will be connected to the mechanical drop system. Determining the exact ratio of static string to bungee cord required a combination of equations, as demonstrated below.

First, one must take into account the total distance that the egg must cover in the drop (h). When the egg is at the bottom of the drop, this distance will be comprised of three different segments: the length of the static line ($L_s$) the length of the upstretched bungee ($L_b$) and the amount that the bungee stretches during the fall (x).

\[ h = L_s + L_b + x \] (3)

Second, one must determine how far the bungee will stretch (the “x” term) given the fall from height “h”. This is best approached through the lens of potential energy. At the bottom of the bungee jump (when $h=0$), the potential energy due to gravity is zero:

\[ PE_g = mgh; h=0 \]

\[ \text{therefore,} \]

\[ PE_g=0 \] (4)

The gravitational potential energy experienced throughout the earlier portions of the jump (when $PE_g \neq 0$) has been stored in the bungee as potential energy. This potential energy can be described in the following two equations (where $k$ is the spring constant and $x$ is the displacement, or “stretch”, of the bungee):

\[ PE_b = (0.5)kx^2 \] (5)
\[ \text{Pe}_b = \text{Pe}_g \text{ (top of drop)} \] (6)

The combination of Equations 4, 5, and 6 yields the following equation, having solved for “x”:

\[ x = \frac{2mgh}{k} \] (7)

Finally, combination of Equations 2, 3 and 7 yields the following equation from which the length of the static line and the length of “stretch” for any given length of bungee cord can be determined:

\[ h = L_s + L_b + \frac{2mghL_b}{0.762} \] (8)

In addition to ensuring that the egg gets as close to the ground as possible, without breaking, a safe bungee design must also take into account the acceleration experienced by the egg throughout the bungee jump. The egg must not experience any acceleration greater than 3g. The egg’s greatest acceleration will occur at the very bottom of the bungee jump. This acceleration can be represented by the following formulas:

\[ a_{\text{total}} = a_{\text{spring}} - a_{\text{gravity}} \]
\[ a_{\text{total}} = \frac{kx}{m} - g \] (9) (10)

Substitution of the “x” value determined in Equation 7 into that above equation will yield the maximum acceleration experienced by the egg. This experimental design presupposes that the experiment will be performed with a set length of bungee. This simplifies calculations by making \( L_b \) a constant rather than another variable to be solved for. Selection of this length is at the experimenter’s discretion, but care should be taken to ensure that the length of selected bungee does not create an acceleration too great for the egg to withstand.

This model was subjected to informal testing through the creation of a small-scale model of the egg drop. This trial was run with the parameters outlined in Table 2 and was run several times. Although no quantitative data was collected regarding the success of failure of this drop, qualitative observations were made to assess the effectiveness of the model. It proved extremely effective at executing the bungee jump without allowing the mass to collide with the floor. All of the runs safely returned the mass to equilibrium without hitting the floor. This was confirmed by the use of a camera to record the mass as it neared the bottom of the jump. The footage of the drop was played back slowly to check for any possible collision. In all cases, the mass did not collide with the floor, but came rather close. This result suggests that use of Equation 8 to determine bungee parameters is likely to be a successful model for designing the final bungee jump.

Error in this experiment could have occurred in many locations. Measuring error when measuring displacement in the first part of the experiment would alter Equation 2, which
would, in turn, alter Equation 8. A mistake in the generation of the bungee jump parameters could prove disastrous for the egg. Additionally, calculations done in the second part of the experiment neglect air drag. Although it did not seem to affect the small-scale model, the introduction of air drag in the actual bungee jump experiment could alter the larger system. Improvements could be made to this experiment through more precise measurements and inclusion of air drag in the final calculations.

V. CONCLUSION

This experiment aimed to create a mathematical model to represent the change in spring constant as the length of the bungee was altered (Equation 2) and, with this information in hand, create and test a small-scale model of the final bungee jump. By analyzing the bungee jump in terms of potential and kinetic energy, a formula to determine the length of static string (relative to the amount of bungee cord) was successfully developed (Equation 8). Small-scale tests of this set-up, and the formula by extension, proved very successful.

VI. APPENDIX

Table 1: Displacement resulting from varying mass for several lengths of bungee and their corresponding “spring” constants.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Weight (N)</th>
<th>Displacement (m)</th>
<th>Spring Constant (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0500±0.0001</td>
<td>0.4900±0.0002</td>
<td>0.248±0.005</td>
<td>1.99±0.05</td>
</tr>
<tr>
<td>0.1000±0.0001</td>
<td>0.9800±0.0002</td>
<td>0.423±0.005</td>
<td>0.9000±0.0002</td>
</tr>
<tr>
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<td>0.522±0.005</td>
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<tr>
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</tr>
<tr>
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<tr>
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<tr>
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Table 2: Parameters of Test Drop

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Total Height of Drop (m)</td>
<td>2.000 ± 0.005</td>
</tr>
<tr>
<td>Mass of Object Being Dropped (kg)</td>
<td>0.2500 ± 0.0001</td>
</tr>
<tr>
<td>Length of Bungee (m)</td>
<td>0.300 ± 0.005</td>
</tr>
<tr>
<td>K Value (Determined by Equation 2) (N/m)</td>
<td>2.5 ± 0.1</td>
</tr>
<tr>
<td>----------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Length of Static String (Determined by Equation 8) (m)</td>
<td>0.18 ± 0.05</td>
</tr>
</tbody>
</table>

Figure 1: Set-up for part one of experiment – this set-up was used to determine the spring constant for various lengths of bungee.

Figure 2: Displacement vs. Weight for 0.377m Bungee

\[ w = 1.9887x + 0.1378 \]
\[ R^2 = 0.99992 \]
Figure 3: Displacement vs. Weight for 0.500m Bungee

**0.500m Bungee**

\[ w = 1.5659x + 0.1309 \]
\[ R^2 = 0.9844 \]

Figure 4: Displacement vs. Weight for 0.615m Bungee

**0.615m Bungee**

\[ w = 1.2302x + 0.2478 \]
\[ R^2 = 0.99983 \]
Figure 5: Displacement vs. Weight for 0.790m Bungee

![Displacement vs. Weight for 0.790m Bungee](image)

\[ w = 0.9582x + 0.2746 \]
\[ R^2 = 0.99508 \]

Figure 6: Spring Constant vs. Length of Bungee

![Spring Constant vs. Length of Bungee](image)

\[ k = 0.762(L_b)^{0.999} \]
\[ R^2 = 0.99631 \]
Figure 7: Final Bungee Jump Set-up