In a system in which a mass is hanging from bungee, how does varying the length of the bungee impact the spring constant (k) of the bungee?

Introduction

In this experiment, we examined the impacts of varying the length of our bungee on the spring constant (k) while the mass is held constant. Our hypothesis is that as length is added to the bungee the spring constant will decrease at a constant rate. Our setup will be a mass of 100 grams hanging at the end of bungee. By studying the impacts of the length of the bungee on the spring constant it will allow us to better understand and use the necessary formulas for our final experiment, the bungee jump. The most important formula in this experiment is Hooke’s law. This equation is of vital importance to this experiment because it allows us to calculate the impacts of our added variables.

\[ F_{spring} = -kX \]  
(Eq. 1)

\( F_{spring} \) = The force of the spring (Newtons)  
\( k \) = spring constant (Newton/ Meters)  
\( X \) = displacement (meter)

Methods

In order to properly track the impacts of added length of bungee on the spring we used the set up below. This setup is optimal and reduces the chances of error.  
A mass of 100 grams was attached to the bottom of the bungee by knotting the end of the bungee in a loop. This mass held constant so that the force in equation 1 is constant. Both the ruler and bungee were attached to rod and both hung from this rod. The ruler is used to calculate that un-stretched distance as well as the equilibrium distance.
Diagram 1

1: Bungee
This is our controlled variable. We will vary the length of the bungee in order to solve our question.

2: Hanging Mass (100 grams)
The mass in this experiment is our constant. It leads us to have a constant force throughout all trials.

3: Ruler
The ruler allows us to measure both the un-stretched length and the equilibrium length of the bungee.

In this experiment we decided to look into the general trend of the spring constant as the length of the bungee increased. We decided to add approximately 8-14 cm of un-stretched bungee to previous length every time, starting at 31.2 cms. In order for the force of the spring to be held constant we kept the hanging mass at a constant mass of 100 grams throughout the entire experiment. By keeping the mass constant at 100 grams it kept the force acting on the system constant, which is the force of gravity. First, we measured the length of the bungee with no mass and then with the 100-gram mass for each length using the ruler hanging directly to the side of it. It is important to attempt to keep the loops, which attaches the mass to the bungee at a very similar size. The loop that attaches the bungee to the top of the system should never change only the bottom loop should be adjusted in order to add length. With the mass of the stretched length and the mass of the un-stretched length we are able to calculate the distance (X) in equation 1. By obtaining this number and obtaining the force (multiply the mass times \(-9.8 m/s^2\)) we are able to solve for K using equation 1. We then repeated this process until we reached approximately 100 cm of un-stretched bungee and recorded the K value for each trial.

Results:

Our results were in accordance with our hypothesis, as the length of the bungee increased the spring constant decreased.

Figure 1: This table consisted of the un-stretched length of the bungee and the value of \(k\). The value of \(k\) was calculated using equation 1 and the un-stretched length was measured by using a ruler.

<table>
<thead>
<tr>
<th>Un-stretched length(cm)</th>
<th>Value of K (N/M)</th>
<th>1/un-stretched length(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.5</td>
<td>0.025</td>
<td>0.021052632</td>
</tr>
<tr>
<td>61.5</td>
<td>0.0196</td>
<td>0.016260163</td>
</tr>
<tr>
<td>73</td>
<td>0.016</td>
<td>0.01369863</td>
</tr>
</tbody>
</table>
This table allowed us to track the trend of the spring constant as we increased the length of the bungee. We controlled the un-stretched length of the bungee and calculated $K$ using Hooke’s law. In order to use Hooke’s law we had to have the displacement of the mass and the force acting on the mass. The force acting on the mass was equivalent to the force of gravity acting on the 100-gram mass, which was .98 Newtons. We then determined the value of $X$ by subtracting the value of the un-stretched length by the value of the equilibrium length, which can be seen in figure 2 below.

Figure 2- This is the value of the un-stretched length of the bungee, the value of the equilibrium length of the bungee after a mass is added and the displacement of the hanging mass.

<table>
<thead>
<tr>
<th>Un-stretched length (cm)</th>
<th>Equilibrium length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61.5</td>
<td>119.5</td>
</tr>
<tr>
<td>73</td>
<td>140.6</td>
</tr>
<tr>
<td>80.8</td>
<td>152</td>
</tr>
<tr>
<td>89.3</td>
<td>166</td>
</tr>
<tr>
<td>100.7</td>
<td>180.5</td>
</tr>
<tr>
<td>48</td>
<td>95.5</td>
</tr>
<tr>
<td>31.2</td>
<td>67.7</td>
</tr>
<tr>
<td>39.8</td>
<td>81.7</td>
</tr>
</tbody>
</table>

We used the values in this table to calculate the different values of $X$. In Hooke’s law the value of $X$ is the displacement of the hanging mass. In our experiment the displacement is equal to the distance between the un-stretched length of the bungee and the equilibrium length after the 100 gram mass is added.

Figure 3: This is the graph of the value of the spring constant vs. the un-stretched length of the bungee.
Through this graph we were able to see the spring constant is inversely proportional to the unstretched length of the bungee. According to the data in this graph $K = 0.7024 (\text{unstretched length of the bungee})^{-0.871}$.

Figure 4: This is the graph of the spring constant and $1/(\text{un-stretched length of the bungee})$.

This graph is extremely important to future experiments regarding our bungee jump. This graph allows us to see the impact of added length to the bungee on the spring constant. Through this graph we are able to see that as $1/\text{un-stretched length}$ increased
(or the un-stretched length decreased) the value of K increased. Our linearized equation that we derived from the graph is $K = 0.986(1/\text{un-stretched length}) - 0.0027$.

Through observation of all of these graphs it is evident that as the un-stretched length of the bungee increases the value of $k$ decreases. In figure 4 the uncertainty of the impact of un-stretched length on the value of $k$ is 3.3% uncertainty. The uncertainty in measurement was .033 cm.

Discussion

This experiment was in agreement with our hypothesis that as the length of the bungee increased the value of the spring constant decreased. From our experiment we were obtained the formula $K = 0.986(1/\text{un-stretched length}) - 0.0027$. This formula allows us to see how different amounts of bungee impacts the value of the spring constant. Throughout the experiment our uncertainty in our measurement was $\pm .0334$ cm.

We determined that there were many areas of potential error in our experiment. Some of the possible areas of error include the way in which we attached the bungee to the mass. The mass was attached by tying a loop with the end of the bungee. This loop changed for each trial and could have impacted our results depending on the different ways and size of the loop. Another source of error is that the bungee elasticity could have changed throughout the experiment. This means that due to the mass hanging on the bungee for too long it may have stretched the bungee out. The impact of these errors is difficult to quantify. I do not believe that these errors had a significant impact on the experiment since our data was in accordance with our hypothesis as well as the fact that our percent uncertainty was only 3.3%. If we were able to retest this experiment I would choose a constant length that the loop would be for each trial. This would greatly reduce the impacts of the loop on the data.

Conclusion

This experiment allowed us to prove that as the length of our bungee increased the value of $k$ decreased. This fact and the equation $K = 0.986(1/\text{un-stretched length}) - 0.0027$ has extremely important implications in later experiments. By understanding the concept and applying our new formula this will prevent our egg from hitting the ground during its bungee jump. Without this formula, we would have not assumed that the $K$ value decreases with added distance to the bungee and our egg would have hit the ground. I believe that there are many ways to add to this experiment. One way in which we can build upon this experiment is by testing how $k$ is impacted by the amount of times the bungee is used per drop. This would allow us make sure that our bungee would not lose its elasticity.