Estimating the Value of K for a Length of a Bungee

**Introduction:**

In order to ensure a successful bungee jump that will not shatter the egg, the maximum stretch of the bungee needs to be estimated. In a previous experiment it was discovered that a bungee does not follow Hooke’s Law. Therefore the law may not be used for estimating maximum stretch. Hooke’s Law is shown in equation 2 below where $F_x$ is the force total, $k$ is the spring constant, and $x$ is the amount displaced. This equation does not work because the bungee has an elastic nature that ideal springs do not possess. *Disclaimer even though the equation may not model bungee behavior perfectly, understanding the approximations of $k$ will assist you with the bungee jump decisions. It is important to determine the first maximum oscillation, because after that the bungee will never stretch pass the maximum distance. This constant ($k$) can be modeled by determining the relationship between the static length of the bungee ($L$) and how much it stretches at that length ($x$). This was modeled by plotting the Force of gravity vs. the displacement of the bungee. This experiment will explore the relationship of $k$ at a certain static length and then this data will then be used to determine the relationship of $k$ and static...
length as the length increases. In other words, the k value at maximum length for a safe jump will be extrapolated from the data received in this small-scale experiment. This will allow a successful model for the maximum static length the bungee cord can be at during the initial 'jump' without the jumper breaking on the ground.

**Equation 1:**
\[ F_g = mg \]

**Equation 2:**
\[ F_x = -kx \]

**Methods:**
Two knots were tied in the standard way as to not damage the bungee cord. These knots were made as small as possible in order to minimize loop stretch under 2cm. The length between the two knots (not including the knots) was then measured flat on the table and recorded as 0.592m in the lab journal. One bungee loop was then looped onto the platform and the other held a platform with a mass of 50g. A tape measure was then hung behind the bungee in order to accurately measure the displacement of the bungee; this was measured by subtracting the length of the unstretched (static) bungee length (L) from the total of the stretched length of the bungee after the weight was added. See Figure 1 for an illustration of the set-up. A total mass of .1kg, .120kg, .130kg, .150kg, and .170kg were measured for the amount

![Figure 1: Depiction of setup. *not to scale](image)
of bungee they displaced. These steps were then repeated for 3 additional static lengths: 0.130m, 0.289m, and 0.470m.

Results:

The graphs of force of gravity and the displacement of the bungee showed a linear increase where increasing force of gravity ($F_g$) equaled a linear increase in displacement ($x$). The slope of this trendline is the $k$ value i.e. $F_g/x$ is equal to $k$ when rearranging Hook’s law, which was then plotted and fitted to a logarithmic relationship between the $k$ constant and the length of the bungee in meters. As the length of the static bungee increased the $k$ value decreased at a constant logarithmic rate.

![Graph of $f_g$ vs. $x$](image)

**Figure 2: Graph of the 4 static lengths of bungee with the displacement at force $f_g$**
The amount of displacement for each static line has been graphed with respect to the amount of force applied (Figure 2). The linear relationships obtained from these relationships reflect the same variable set up as Equation 2; therefore, the coefficients for x in the linear regressions are the k value. Here, k is the spring constant at a certain length and x is the amount of displacement in meters. Thus, as the $f_g$ is increased the displacement is always increased, creating a positive relationship between the two variables. The K constants for each of the lengths are as follows.

Table 2: The standard error values for each equation found by running regression in excel

<table>
<thead>
<tr>
<th>Static length of bungee (m) $\pm 0.005$</th>
<th>Spring constant k (N/m$\pm.046$ N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.130</td>
<td>9.631</td>
</tr>
<tr>
<td>0.289</td>
<td>5.031</td>
</tr>
<tr>
<td>0.470</td>
<td>2.700</td>
</tr>
<tr>
<td>0.592</td>
<td>1.198</td>
</tr>
</tbody>
</table>

Table 2: K constants gathered
A detailed analysis of one bungee length is provided below in order to provide a more detailed view of this process.

\[ F_g = 9.6307x \]

Where 9.6307 is equal to the k value at this (0.130m) constant length of bungee cord. The standard error was run through the regression function in Excel and shows a 0.341 m error for this particular line. Provide percent error (std./average)

All of the k values (Table 1) were then plotted onto a graph in order to see the relationship between the k value and the static length of the cord.

**Length constant and varying mass for 1 bungee 0.130 m long**

<table>
<thead>
<tr>
<th>( f_g (N) ) ±0.046</th>
<th>( m ) (kg) ±0.001</th>
<th>( x ) ±0.005</th>
<th>( \text{Standard Error} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.981</td>
<td>0.100</td>
<td>0.038</td>
<td></td>
</tr>
<tr>
<td>1.18</td>
<td>0.120</td>
<td>0.056</td>
<td></td>
</tr>
<tr>
<td>1.28</td>
<td>0.130</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>1.47</td>
<td>0.150</td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td>1.67</td>
<td>0.170</td>
<td>0.109</td>
<td>0.341</td>
</tr>
</tbody>
</table>

\( F_g \) is equal to the mass \( (m) \) multiplied by the gravity constant and \( x \) is the displacement measured in meters. There is a positive, linear relationship between \( F_g \) applied and the displacement of the bungee.

**Equation 3:** \( F_g = 9.6307x \)
As the Static length of the cord increases the value of K at that length goes down at a constant logarithmic rate. This logarithmic line was fitted to the graph with only 0.008 standard error using the regression function in excel. This gives the equation:

**Equation 4:**

\[ k = -5.489\ln(x) - 1.6181 \]

Where \( k \) is the spring constant for the bungee at the natural log of the static length of the cord.

**Table 4: Standard error for the function**

<table>
<thead>
<tr>
<th>X Variable 1</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.008</td>
</tr>
</tbody>
</table>

**Discussion:**

The overall result of this experiment was to see if the \( k \) value for a certain length of bungee could be modeled. These results can be used to model different \( k \)
values for different lengths of the bungee cord in accordance with equation 3. By understanding the relationship between k and a static length, the maximum static length of the cord can be modeled and thus insure a successful jump. This will allow the researcher to estimate the maximum stretch of the cord for the initial jump.

Even though the bungee cord is not representative of Hooke’s Law, the bungee will not stretch pass this initial maximum displacement. By being able to model the K value of the bungee we can then use the total energy (Kinetic energy and gravitational potential energy) to estimate the amount of stretch the bungee will undergo in accordance with the f_s of the egg. This is possible because of the law of conservation of energy that states that energy must be preserved within a system. Gravity is preserved and we can approximate the bungee force to be so. Since the egg is aerodynamic its air resistance can be ignored in these calculations.¹

The logarithmic relationship was the best fitting line with an uncertainty of only 1%; however, due to the complexities involved with manipulating logarithmic function, one might want to use a power function for simplicity despite its higher percent error. The human error associated with measuring displacements and static length comes from being unable to distinguish the minute differences in the tape measure. This could be better resolved by having multiple people view the lengths. The error could also change depending on the knots tied, if a different knot is used it could slightly affect the k value of the bungee. In addition the loops around the actual egg could be great enough to produce additional stretch and thus making the final stretch slightly longer than the stretch modeled. To minimize these errors

¹ Professor Cook in discussion in class
two people should double check the length of the cords the stretch of the loops should be evaluated. Either reviewing another experiment or running a quick experiment before attempting the bungee jump can allow the researcher to determine the loop stretch relationship. Systematic error would include the elastic nature of the bungees. If a bungee has been over-stretched then its k value will differ substantially from this projected value. In addition the masses tested were not double-checked on a scale and could also be slightly different than the written values. These can be fixed by double-checking the mass of the attached weights and not allowing the cord to be stretched for long periods of time.

**Conclusion:**

This experiment shows a method for estimating the k value of the bungee at a certain length. This would enable us to model the length of bungee one needs to use in order to ensure a successful jump. In further research, we should use the formula KE=.5KX^2 and the conservation of energy theorem to further test if our experimentally determined k values remain useful. We recommend that the bungee challenge participants use these equations in conjunction with our found k relationship when choosing bungee lengths for a successful jump.