“Finding K Through Energy Transfers”

Introduction:
In this experiment, we looked at the relationship between the unstretched cord length and the k-constant of the bungee cord. To do this, we dropped a 50 gram mass from a certain height, $h$, and recorded the distance it stretched, $x$. Under the assumption that no energy was lost, we were able to use potential energy and elastic energy equations to calculate the k-constant of the bungee cord at various lengths.

Equation 1: $\text{Energy Potential} = mgh$
Equation 2: $\text{Energy Elastic} = \frac{1}{2}kx^2$

By assuming that no energy is lost and that the bungee cord will behave like an ideal spring, we are allowed to say that all the potential energy transfers into elastic energy due to the concept of the conservation of energy. Knowing the numerical values of $m$, $g$, $h$, and $x$, we can isolate and solve for $k$.

Equation 3: $mgh = \frac{1}{2}kx^2$

The research question my partner and I established was: what is the k-constant of the bungee cord—given the mass, the acceleration due to gravity, $g$, the change in height, $h$, and the distance the cord stretches, $x$—for various lengths of the bungee cord?

Methods:
The structure of our experiment was to have a 50 gram mass, which was attached to one end of our bungee cord, fall from a certain height. From the fall, we measured the stretch of the cord and the change in the height.

Diagram 1:
Unstretched Cord:

Object at top (max height):

Object at bottom (min height):

In this diagram, the three different stages of the cord are displayed to best exemplify how the experiment was conducted. In the image on the far left, the cord hangs with no mass. Assuming that there is no additional stretch from its own weight, this was the value we took to be its unstretched cord length. In the middle image, the mass is held at the very top. At this point, all of its energy is in the form of
potential energy. In the image on the far right, the mass has been dropped, allowing the potential energy to transfer into elastic energy. The independent variable and dependent variable are shown to best demonstrate how these values were obtained.

A steel rod, placed perfectly vertical at the end of a table, held the system of the cord and mass up. Attaching one end of the cord to the top of the rod and the other end to the hook placed on the hanging mass, my partner and I were able to perform a repeatable experiment for which to collect our data.

By recording the drops on an iPad, the data from the experiment was more accurate as the videos recorded allowed us to review the fall in a slower, more controlled manner from which we were able to obtain our results. Additionally, having placed a meter stick next to the bungee cord, we were able to measure the height change during the different segments of the fall as well as the stretch of the bungee cord at the bottom of the fall very precisely.

The independent variable in the experiment was the unstretched cord length, measured in meters, and the dependent variable was the stretch of the cord, also in meters. We were able to determine the change in height from the raw data we collected. The value for the change in height was the unstretched cord length plus the length stretched, or the displacement of the mass. Knowing this, we were able to calculate the stretch of the cord, \( x \), by subtracting the height \( h \), from the displacement of the mass, \( \Delta h \).

Following these procedures for four different cord lengths (1.30m, 0.82m, 0.63m, and 0.31m) of the unstretched cord, we were able to determine the changing values for the k-constant for various cord lengths.

To minimize the effects of confounding variables between the various IV trials, the hanging mass was dropped from the same distance from the ground each time and was dropped in the same manner each time—not at an angle and without any downward force. By keeping this system for data collection consistent, the impact from random error is reduced and the results are more accurate. It is especially important to note that the mass was simply dropped and not thrown downward, as this would impact the initial energy by adding a starting kinetic energy value.

**Results:**

To ensure that the data collected was all reasonable, my partner and I did five trials for each of the four lengths, obtaining five values for the change in height and the distance stretched for each independent variable. For uncertainty measurements, we used a measurement uncertainty of +/- 0.005 m for all lengths measured, including the unstretched cord length. We felt that this was fairly reasonable as the markings on the ruler used were precise enough to allow us to measure to half of a centimeter.

<table>
<thead>
<tr>
<th>Table 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Change in height “Displacement” (m)</strong></td>
</tr>
<tr>
<td>Trial 1</td>
</tr>
<tr>
<td>Trial 2</td>
</tr>
<tr>
<td>Trial 3</td>
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<tr>
<td>Trial 4</td>
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<tr>
<td>Trial 5</td>
</tr>
</tbody>
</table>

Table 1 shows the results for the displacement of the hanging mass for each of the five trials done for the four unstretched cord lengths (0.310m, 0.630m, 0.820m, and 1.300m). The displacement was defined as the full distance the mass fell from its highest position to its lowest position—shown in Diagram 1 as the “change in height.”
Table 2:

<table>
<thead>
<tr>
<th>Distance Stretched (m)</th>
<th>Unstretched Cord Length of 0.310 (m) +/-0.007 m</th>
<th>Unstretched Cord Length of 0.630 (m) +/-0.007 m</th>
<th>Unstretched Cord Length of 0.820 (m) +/-0.007 m</th>
<th>Unstretched Cord Length of 1.300 (m) +/-0.007 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 1</td>
<td>0.445</td>
<td>0.765</td>
<td>1.010</td>
<td>1.385</td>
</tr>
<tr>
<td>Trial 2</td>
<td>0.435</td>
<td>0.780</td>
<td>1.020</td>
<td>1.375</td>
</tr>
<tr>
<td>Trial 3</td>
<td>0.430</td>
<td>0.780</td>
<td>1.030</td>
<td>1.385</td>
</tr>
<tr>
<td>Trial 4</td>
<td>0.445</td>
<td>0.775</td>
<td>1.030</td>
<td>1.380</td>
</tr>
<tr>
<td>Trial 5</td>
<td>0.445</td>
<td>0.775</td>
<td>1.040</td>
<td>1.370</td>
</tr>
</tbody>
</table>

Table 2 demonstrates the value for $x$, the dependent variable of the experiment. This was the “change in height” minus the unstretched cord length. The uncertainty for Table 2 was found by using the “propagation of uncertainty in sums or differences” method because there was uncertainty in both the unstretched cord length and the displacement—which were used to calculate the distance stretched, as seen in Equation 4 below.

Equation 4: $Distance \ Stretched \ x = Displacement \ \Delta h - Unstretched \ Length \ of \ Cord \ (h)$

In Equation 4, all measurements are made in meters and the variables $x, \Delta h,$ and $h$ are all defined in Diagram 1.

The next step was to find the average distance stretched and the average change in height for each of the different cord lengths. The results are shown below in Table 3.

Table 3:

<table>
<thead>
<tr>
<th>Unstretched Cord Length (m) +/- 0.005</th>
<th>Average Change in Height (m) +/- 0.005 m</th>
<th>Average Distance Stretched (m) +/- 0.007 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>0.750</td>
<td>0.440</td>
</tr>
<tr>
<td>0.63</td>
<td>1.405</td>
<td>0.775</td>
</tr>
<tr>
<td>0.82</td>
<td>1.846</td>
<td>1.026</td>
</tr>
<tr>
<td>1.30</td>
<td>2.679</td>
<td>1.379</td>
</tr>
</tbody>
</table>

From here, we were able to calculate the change in potential energy of the system by multiplying the change in height by the mass, 50 grams, by the acceleration due to gravity, $9.81 \frac{m}{s^2}$. Since the weight was pre-measured to be 50 grams—which we re-measured three times and received the same value—we decided to declare the uncertainty on this measurement negligible due to the weakest link rule. Additionally, the acceleration due to gravity does not require an uncertainty, leaving the uncertainty to be defined by the lengths measured.

By comparing the results for potential energy to the square of the distance stretched, we were able to calculate $k$ by using Equation 5. The uncertainties below were found using the “propagation of uncertainty of a product or quotient” method. The results are recorded in Table 4, with the values for the potential energy and average distance stretched squared included beside the k-constant values for clarity.

Equation 5: $\frac{2mg\Delta h}{x^2} = k$
By graphing the calculated k-constant value against the unstretched cord length in Graph 1, a clear pattern appears.

Graph 1:

![Graph 1: K-Constant Vs. Unstretched Cord Length](image)

Graphing the k-constant against the length of the cord produced an inverse relationship. To analyze this more appropriately, a linearized version of the graph is required. To linearize the graph, one axis must be the calculated k-constant values and the second axis must be 1/L—one over the unstretched cord length (\(\frac{1}{m}\)). This is represented in Graph 2.
Graph 2:

\[ y = 0.9962x + 0.6035 \]

From this graph, we received the equation:

Equation 6:

\[ K \text{ constant } \frac{N}{m} = 0.9962 N \pm 0.0555 \frac{N}{m} \left( \text{Unstretched cord length} \right)^{-1} + 0.6035 \frac{N}{m} \pm 0.1074 \frac{N}{m} \]

By linearizing the data, we can better fine the uncertainty in the slope and discover that there is a y-intercept by separating the initial k-constant value, the y-intercept, from the change in the k-constant as the x-value varies, the slope. This is important as it states that the cord is not an ideal spring as it should not have any value for the y-intercept—it is physically impossible that at 0m in cord length there is a k-constant.

The final results produced are found in Table 5 below.

<table>
<thead>
<tr>
<th>Unstretched Cord Length (m) +/- 0.005</th>
<th>Calculated K-Constant ( \frac{N}{m} ) +/- 0.08 ( \frac{N}{m} )</th>
<th>Expected K-Constant ( \frac{N}{m} ) *from Trendline +/- 0.1629 ( \frac{N}{m} )</th>
<th>Percent Difference [ 100% \times \frac{K_{\text{calculated}} - K_{\text{expected}}}{K_{\text{average}}} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.310</td>
<td>3.80 +/- 2%</td>
<td>3.79 +/- 4%</td>
<td>0.3%</td>
</tr>
<tr>
<td>0.630</td>
<td>2.29 +/- 4%</td>
<td>2.20 +/- 7%</td>
<td>4%</td>
</tr>
<tr>
<td>0.820</td>
<td>1.72 +/- 5%</td>
<td>1.80 +/- 9%</td>
<td>5%</td>
</tr>
<tr>
<td>1.300</td>
<td>1.38 +/- 6%</td>
<td>1.40 +/- 11%</td>
<td>2%</td>
</tr>
</tbody>
</table>

In Table 5, the first column is the unstretched cord length, followed by the calculated K-constant values and corresponding percent uncertainty. These calculated values are produced based off of the energy formulas, as shown in Equation 5, and are identical to the K-constant values found in Table 4. The next column over is the expected K-constant value and percent uncertainty. This is produced from Graph 2’s trendline. By comparing these two values for the K-constant at each unstretched cord length, we can see how well the trendline predicts the actual K-constant values the experiment seeks to find. This comparison is quantified in the final column with the percent error. Because the percent difference is
smaller than the percent uncertainty, the results produced by Graph 2’s trendline are deemed as acceptable predictions for the K-constant of the bungee cord.

**Discussion:**

Equation 6 allows us to predict the k-constant for any length of the cord when it is not stretched. Theoretically, the slope for this equation should be 1 N, as the k-constant value should be directly proportional to the inverse of the length on the cord. This allowed us to find the percent uncertainty for the data.

\[ 1 \text{ N} - 0.9962 \frac{N}{1} \times 100\% = 0.4\% \text{ Percent Error} \]

Additionally, we can compare these results to the results we received in the first Bungee Cord Lab. In the first lab—for which my partner and I found the value for the k-constant through Hooke’s Law (shown in Equation 7) in a static experiment—we found a k-constant v. inverse length slope of 0.9620 N.

Equation 7: \( F = -kx \)

\[ 0.9962 N - 0.9620 \frac{N}{1} \times 100\% = 3.5\% \text{ Percent Difference} \]

Neither the percent error nor the percent difference in the slope seem to be too drastically different from the results we found in this lab, allowing us to feel confident in the results we achieved. Comparing these values to the percent uncertainty, 5.6%, one sees that the percent error and percent difference in the slope is smaller—meaning that the slope this lab produced is accurate.

\[ 0.0555 N \times 100\% = 5.6\% \text{ Percent Uncertainty} \]

Comparing these results to our current lab, my partner and I found a difference of 3.5% for the slope, however, comparing individual values found for the k-constant at the smaller unstretched cord lengths leads to larger differences. For example, at the unstretched cord length of 0.310 m, the percent difference is about seven and a half times larger than the percent difference of the slope.

\[ \left( \frac{3.8 N}{m} - 2.9 \frac{N}{m} \right) \times 100\% = 26.9\% \]

This is due, in large part, to the drastic differences in the y-intercept. The y-intercept also created a larger percent uncertainty for the expected K-constant values found in Table 5 as the uncertainty from the y-intercept dominated the overall uncertainty.

One possible reason for the y-intercept includes changes in the stretch of the cord due to hegemony—the product of stretching a newer spring or cord, changing its k-constant as it “loosens up.” This is a very important source of systematic error that would likely have an impact on both the slope and the y-intercept as it not only changes the data between trials, but also between the different lengths of the cord. Additionally, it changed the results between the two experiments—the previous Bungee Lab and the current Bungee Lab.
Additionally, the control measurements of the unstretched cord length as it hung with no mass (as shown in Diagram 1 in the far left image) may have been affected by the non-zero weight of the cord pulling on itself. This is especially important as the weight is not pulling evenly throughout the cord as, towards the upper end of the cord, the cord has a tension force equivalent to the weight of a larger percentage of the cord, whereas towards the lower end of the cord, the percentage of the cord, and resulting weight, is greatly reduced. This impact is larger for the longer unstretched cord lengths for this same reason.

Other sources of random and systematic error include the different sizes of the loops tied in the cord used to hang the masses from the cord and any measurement inaccuracies. It was fairly difficult to obtain precise and accurate data for the unstretched cord length at times as, by pulling the cord straight, we would stretch the cord, however, leaving it loose made it curve. Additionally, the stretched cord length could have inconsistencies due to the hegemony of the cord. Another consideration is that the hanging mass would fall the full length of the unstretched cord length before actually stretching the cord. Depending on the length of the cord, this may have affected the force from tension the cord experiences. There is no way to improve on this, however, as it is necessary to gather varied data in terms of the cord length.

To aid our efforts, we recorded the falls with the iPad, however, sometimes, it seemed as if the iPad’s angle was not actually parallel to the fall and ruler. This slight slant caused an angled reading of the length. As for the differences in loop sizes, the loops we tied were consistently small. Due to this fact, we are comfortable in saying that this had a very small effect, if any, on our data.

The impact from the way in which my partner and I measured the length of the unstretched cord may have had a more significant impact on the results as it would create a larger value for the cord length than it should otherwise have been as it was not completely unstretched. This would change the values for the distance stretched, \( x \), and the displacement, \( \Delta h \), to be smaller than they actually are—affecting all the calculations that used these values.

Finally, the changes due to hegemony seem to have the greatest impact. Throughout the experiment, my partner and I noticed changes in the length of the cord when it was no longer stretched. We also noticed that it was harder for us to stretch the cord the same distance. This seems to align with the data we have because, as the cord was stretched more it would become longer in its unstretched state. Since the cord can only stretch to a certain maximum length without breaking, it was harder to stretch the same distance, making the value for the k-constant increase. This can be illustrated in the math below.

\[
M = \text{Max length the cord can reach when stretched} \\
L_0 = \text{Original unstretched cord length} \\
L = \text{Change in cord length after being stretched} \\
x_o = \text{Original distance stretched} \\
x_f = \text{Final distance stretched} \\
M = L_0 + x_o \\
M = L_0 + L + x_f
\]

Mathematically, this would suggest that \( x_f \) is smaller than \( x_o \).

According to Equation 7, if the value for \( x \) stays the same while the value for \( F \) increases, then the value for \( k \) will also increase. Not only does this help explain the differences between the previous lab and this lab, but it can also affect the results during the lab as the cord becomes more and more stretched out.

Ways to minimize the sources of errors above include limiting the time the cord is stretched and the weight hung from the cord or stretching the cord thoroughly prior to conducting the experiment. This will reduce the effects of hegemony by either letting it remain not stretched out or stretched out—keeping it consistent rather than having it change during the experiment. Another consideration is to hook the
hanging mass on the cord with tape or some other external method. This will prevent the effects of the hoop stretching and its impact on the k-constant found. Finally, either increasing the distance between the iPad and the point at which the mass is dropped or fixing the iPad in such a position that is perfectly parallel to the fall, will allow for better readings. Keeping the iPad perfectly parallel to the precise point the hanging mass falls is difficult because if it falls slightly outside of the expected range, then the reading is not exact, so increasing the distance between the fall and the iPad can aid this effort by reducing the angle the iPad camera creates.

To minimize the effects of the weight of the cord pulling itself down and stretching while measuring its length, one can measure the cord when it lays flat on the ground. This would eliminate the possibility of the cord stretching, however, the cord does not lie perfectly straight without being taut—meaning the experimenters can either apply some force to the cord and stretch it slightly, or they can measure a slightly shorter length and accept the error from the notstraight unstretched cord. Knowing this difficulty, my partner and I decided the best course of action was to accept the error from the force of gravity slightly stretching the cord. It seemed to produce the most reliable results, especially since the cord during the experiment would be in the same position, hanging from the steel rod. For this reason, eliminating this source of error is not possible.

Overall, the results we achieved make sense according to our empirical knowledge regarding the properties of the cord and allow us to better understand and predict the changing properties of the cord according to its various lengths.

**Conclusion:**

In conclusion, the data we collected in this experiment shows that the unstretched cord length is inversely proportionally to the k-constant. For any change in one over the length of the unstretched cord $(1/m)$, the k-constant increases by $0.9962 \frac{N}{m}$. With evidence from our data, as well as our observations, to support this, we feel confident in this relationship. The $0.38\%$ percent error in the slope of our equation and the $3.6\%$ percent difference between the slopes of the equations found in the two bungee labs, we feel strongly that this lab produced good results.

Seeing how the k-constant values were different in the static and dynamic experiments for the two Bungee Labs, leads to an interesting conclusion that my partner and I will have to carefully select which k-constant values to use in the final Bungee Challenge. The final challenge is dynamic, therefore, it would make this current lab seem more relevant. Additionally, because this dynamic lab is more current, it is believable that the cord, not actually being ideal, might have undergone some changes—such as hegemony—which changed its properties, including the k-constant values.