Finding the Force of the Bungee Cord on the Egg

Introduction

This is the second part of our series of experiments in trying to determine the ideal length of our bungee cord. An ideal length of the bungee cord will allow the egg to fall freely for the maximum amount of time and still stop the egg from breaking from either the cord decelerating it too quickly or smashing on the floor. We will do this using our approximation of its spring constant value from the first set of experiments and our newest results from the second series, in which we measured how different lengths of the bungee cord react to different forces acting on it. The equations we’ll use are

(eq1) \[ \Sigma F = MA \]

(eq2) \[ F_s = -K\Delta Y \]

where \( \Sigma F \) is the sum of the forces acting on the egg, \( M \) is the mass of the egg, \( A \) is the acceleration (or deceleration) of the egg, \( F_s \) is the force of the bungee acting on the egg, \( \Delta Y \) is the amount the bungee stretches and \( K \) is our approximate spring constant. Since we’ll know the mass of the egg and the max acceleration it can reach without breaking (three times the acceleration due to gravity), we can use our the results from this series of experiments to find the \( F_s \) and then have only one unknown variable (\( \Delta Y \)) left to calculate before we can determine the ideal length of the bungee.

Method

To find out how the bungee stretched at different lengths with different forces acting on it, we hung a measured length of it from a stand and then attached several masses of known weight to it with a mass hanger and a knot in the bungee cord. We then taped a tape measurer to the stand right next to the bungee cord and played it out so that it was hanging parallel next to the cord. We would then lift the masses a measured distance of 9cm above the point where the bungee would first start acting on the mass (the point where the bungee would hang without the weight of the mass hanging on it) and then drop them. We used an iPad to record the fall in slow motion so we could measure the total amount that the bungee stretched by looking at where the knot stopped moving against the tape measurer (we also put a white back-drop behind the bungee cord and the tape measurer to make it easier to see these measurements). We used six different lengths of the bungee cord, 10cm, 25cm, 37.5cm, 50cm, 62.5cm and 70cm in our experiment. We then tested the four masses 50g, 60g, 70g and 80g on each different length. We took three different measurements for each of these masses at each different length so that we could calculate an accurate average of the amount the bungee stretched.
Results

After measuring the displacement three times for each mass, we calculated the average displacement they caused on the bungee. We then used eq2 and our calculated k value from our previous experiment (k=1.3077*Equilibrium length of bungee) to calculate the force of the bungee cord acting on the mass.

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<thead>
<tr>
<th>Hanging Mass (kg)</th>
<th>Weight of mass (N)</th>
<th>Average Displacement (m)</th>
<th>Standard Deviation of Displacement</th>
<th>Force of bungee cord on mass (N)</th>
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<tbody>
<tr>
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In order to try and predict the displacement caused by the same forces at different lengths of bungee cord, we graphed the length of the bungee at equilibrium against the force of the bungee on the mass, for each mass.
50g Fs vs Length

\[ y = 0.7693x^{1.8814} \]
\[ R^2 = 0.9999 \]

60g Fs vs Length

\[ y = 0.9375x^{1.8353} \]
\[ R^2 = 0.9986 \]
70g Fs vs Length

Equation: \( y = 1.2067x^{1.8725} \)

\[ R^2 = 1 \]

80g Fs vs Length

Equation: \( y = 1.491x^{1.3969} \)

\[ R^2 = 0.9999 \]
When looking at these results, we noticed that they were all power curves with the displacement values (on the x axis) raised to similar exponents, which indicates a relationship between how long the bungee was and how much force it exerts on the mass. We then used calculated our average that the power curves were raised to be 1.8715 and raised our calculated forces of the bungee on the mass to the negative exponent of the power graph in order to linearize the graphs.

50g linearized 1

\[ y = 127.32x^{-1.6495} \]

60g linearized 1

\[ y = 157.51x^{-1.6329} \]
Unfortunately, this didn’t linearize our graphs, but we did notice that there was another similar relationship between the exponents of these power curves and once again calculated the average exponent to be 1.63315 which we then used as an exponent to raise the equilibrium length of the bungee cord to. This succeeded in linearizing the graphs.
50g linearized

$y = 126.96x - 0.2839$

60g linearized

$y = 134.37x + 0.796$
70g linearized

\[ y = 153.97x + 0.4806 \]

80g linearized

\[ y = 184.04x - 1.385 \]
These results show that there is a direct relationship between the Force of the bungee on the mass raised to the power of -1.8715 and the equilibrium length of the bungee raised to the power of 1.63315.

**Discussion**

Our results show that there is a clear power relationship between the dynamic force of the bungee acting on the mass and the length of the bungee at its equilibrium point. After linearizing the graphs, we will be able to use our established relationship between the $F_s$ and the stretch of the bungee at any given length of the cord. Though this is not the relationship I was expecting to find when we first started this experiment, it is nevertheless very useful as we can use it to find the ideal length of the un-stretched bungee cord.

The uncertainties of our measurements were ±0.3cm for the length of the bungee cord and ±0.1g for our masses. All of our other uncertainties are base off of these and because all of our data is experimental, we do not have any error in our calculations (yet). The main source of error would be measuring the displacement of the knot as the camera on the iPad often blurred the moving knot.

**Conclusion**

Looking back on our methods, it might have been easier to use a pasco force measurer on the falling mass but our experiment was still conclusive and gave us a relationship we can work with. Because our experimental K value and the force of the bungee cord are both related to the total length of the bungee, we'll be able to calculate the ideal length of our bungee cord once we use find the force of the bungee acting on the egg when we use eq1 and eq2 to calculate how much the bungee is going to stretch after it starts acting on the egg.