Application of Hooke’s Law into Real World Bungee

Introduction:

Our goal is to test to see how well our bungee fits Hooke’s Law of an ideal spring:

\[ F = -Kx \]

In Equation 1, F is represented by the force of gravity \( mg \), (m represents the mass in kg of the hanging weight, \( g \) represents the acceleration due to gravity). \( X \) represents the change in distance from equilibrium to stretch point measured in meters, and \( k \) represents the bungee constant. By varying the Force through adding less mass, we can solve for \( K \) in the system. If our bungee follows Hooke’s Law of an ideal spring, then our graph of should find a linear value of \( K \), because \( x \) is proportional to \( F \) in equation 1 so that \( K \) remains constant.

Methods:

Our method was to attach different hanging masses to the bungee cord and record the equilibrium position to find if our bungee followed Hooke’s Law over multiple trials. Also we tested five lengths of un-stretched bungee to see if the bungee constant \( K \) remains the same or changed when testing different lengths of un-stretched bungee cord.
**Figure 1:** Hanging Bungee Set-up. Used our frame of reference with Unit $\hat{i}$ facing upwards. The black in figure 1 represents the bungee, while the blue and orange represent the masses. The Gold is the tape measure while the gray figures represent the pieces that hold up the bungee and the other parts of the experiment.

We used a tape measure, a pole with a clamp to the desk, a pole extension, our bungee cord, a hanging mass, varying masses, and tape. Inside our physics lab, we attached the pole to a lab desk and extended the pole to its highest height of about 2.1 meters so we were able to add higher masses to the bungee. Using the extension off the side of the pole, we taped the top of the tape measure to the pole extension and pulled the tape measure to the ground so the tape measure was taut and did not
sway. Right next to the tape measure on the pole extension, we formed an impermanent knot in the middle of the bungee and attached that to the pole extension. On one end of the bungee cord, we created another impermanent knot to where we attached the hanging mass, and the other end of the bungee cord we pushed off to the side. Our range of masses varied from 50 grams to 200 grams, however for some lengths we had to use smaller masses to make sure the hanging mass did not touch the ground in our given height constraint of roughly 2.1 meters. We used the un-stretched lengths of 27.5 cm, 45.4 cm, 56.1 cm 74.6 cm, and 97.6 cm.

Procedure: Before attaching the bungee cord to the extension of the pole, we pre-stretched the bungee cords to make sure that the bungees would a have consistent stretch capacity. Also when we placed mass on the bungee, we minimized the amount of time under tension to maintain the same elasticity in the bungee. We started by finding the distance of the un-stretched bungee cord with no weight attached, measuring to the bottom of the bungee’s bottom loop. After recording the length, we placed a mass on the end of the bungee and recorded the change in length of the bungee from the un-stretched length to the length at the equilibrium point. The difference in the un-stretched length to the length at equilibrium represents the x is Hooke’s Law (Equation 1). We then repeated this step of recording the change in the length of the bungee for a successively higher mass until we had at least five different masses with respective lengths. By having the tape measure right next to the bungee, we easily could record the length of the bungee to the bottom of the bottom loop. After recording the data for the first un-stretched length of bungee, we untied the middle knot of the bungee and created a new loop for a different un-
stretched bungee length. We completed this process five times, going through the mass variation steps listed above for each length of bungee cord, finding masses to their respective displacement of the bungee in meters.

Results:

From the experiments, we found that the Force and Displacement had a proportional relationship, therefore making the K-value a linear slope with a positive y-intercept. Also as the length of the cord increased, the K-value decreased by an almost inversely proportional relationship.

**Figure 2:** Table of Mass to Displacement. Our first data points for the un-stretched length of 56.1 cm. After recording the Length, we calculated the Displacement of bottom of the Bungee by subtracting the stretched length by the length of the un-stretched bungee.

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Length (cm)</th>
<th>Displacement (x)(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 ± 0.1</td>
<td>70.5 ± 0.5</td>
<td>14.4 ± 0.5</td>
</tr>
<tr>
<td>70 ± 0.1</td>
<td>79.3 ± 0.5</td>
<td>23.2 ± 0.5</td>
</tr>
<tr>
<td>100 ± 0.1</td>
<td>97.7 ± 0.5</td>
<td>41.6 ± 0.5</td>
</tr>
<tr>
<td>120 ± 0.1</td>
<td>112 ± 0.5</td>
<td>55.9 ± 0.5</td>
</tr>
<tr>
<td>150 ± 0.1</td>
<td>138.5 ± 0.5</td>
<td>82.4 ± 0.5</td>
</tr>
<tr>
<td>170 ± 0.1</td>
<td>155.5 ± 0.5</td>
<td>99.4 ± 0.5</td>
</tr>
<tr>
<td>200 ± 0.1</td>
<td>178.1 ± 0.5</td>
<td>122 ± 0.5</td>
</tr>
</tbody>
</table>

We used this original data to calculate the Equation 1 by converting the mass into kilograms and then the displacement from centimeters into meters. The mass and the displacement in Figure 2 have a positive correlation, because as the mass
increases, the displacement increases too. The recorded length’s all have an uncertainty of 0.5 centimeters. The uncertainty in the mass was 0.1 grams, so our uncertainty for the data is 0.5.

**Figure 3:** Finding linear K-Values and Force intercepts of Trend lines. The slope of the trend lines represents the K-Value at a given length of cord. Because of the weight of the system points downward, the negative force cancels the negative sign present in Equation 1. The purple line represents the data from the un-stretched length of 27.5 cm, the navy blue line represents the data from the un-stretched length of 45.4 cm, the red line represents data from the un-stretched length of 56.1 cm, the green line represents the data from the un-stretched length of 74.6 cm, and the light blue line represents the data from the un-stretched length of 97.6 cm.

This graph goes along Hooke’s Law equation (Equation 1), where the force is represented by the weight from the mass on the hanging mass multiplied by
acceleration due to gravity, k is the slope of the trend lines, and x is the
displacement of the bungee because of the added mass in meters. Each trend line
represents a different set of data points for a different un-stretched bungee cord
length. Because each trend line had a multiple points, our uncertainty for the points
in the line is 1 cm. Each of the trend lines has a different slope, however they all
seem to approach a similar nonzero Force intercept instead of passing through the
origin. Because each trend line had a multiple points, our uncertainty for the points
in the line is 1 cm. The average intercept for force axis is 0.345 Newtons with a
standard deviation between the five trend lines of 0.0676. So our uncertainty for the
Force intercepts was 20%.

**Figure 4:** Discovering the relationship between the K-value and Length of the Cord.
The data points in the graph represent the five different K-values (slopes of trend
lines in Figure 3) to their respective un-stretched length of bungee cord.

The relationship between the between the K-Value and the length of the un-
stretched cord is best fit by a negative power function. As the length of the un-

![Graph of Relationship between K-Value and Length of Cord](image)
stretched cord increases the K-Value for the bungee decreases. Because the
relationship between the K-value and the length of the cord is almost an inverse
relationship, we can make the graph relatively linear by finding the relationship of
K-Value to the inverted length of the cord. The uncertainty remains at 20% because
the graph maintains the uncertainty from the figure 3.

**Figure 5:** Linearized Graph of relationship between K-value and Length of Cord. By
inverting the Length of the Cord, the relationship between K-value and 1/Length of
Cord becomes relatively linear.

The graph explains as the K-value increases by 0.8027 for every 1 increase the
inverse in the Length of Cord. The graph fits the line and also goes through the
origin, so the trend line has a y-intercept of zero. So when calculating another K-
value, we can just base the K-value on 0.8027 times the inverse of the length of cord.
Because the inverse of the length of cord is not a perfectly linear line, our
uncertainty rose to 30% uncertainty with this linearized graph.
Discussion:

Our bungee fits Hooke’s Law because the K-values were linear in Figure 3, however it does not follow Hooke’s Law of an ideal spring because of the existence of the Force intercept in the five trend lines. This Force intercept represents the difference between our real world bungee K-value and the ideal spring represented in theory in Hooke’s Law. An ideal spring would have a force intercept of zero. Because there is no accepted value for the force intercept in a real life Hooke’s Law function other than zero, I can estimate the percent error by finding the largest estimate of error in the Force intercept by finding the percent difference between the average Force intercept (0.345), and the smallest Force intercept (0.231) from Figure 3. The percent difference of the force intercepts is 39.6% error. Because there is no known value for percent error, this estimation of percent error through the different is quite high, and may be incorrect because the intercept at 0.231 may be an outlier in the data.

Our sources of uncertainty came from measurements of the exact bottom of the bungee loop. There was about a 0.5cm uncertainty for each length measurement because of the oscillations of the hanging mass. In addition, the masses of the hanging masses we know have least count at 1 grams. Also to measure the length of the bungee without it curling up, we had to pull on the end of the bungee a little bit to make sure the bungee was taut, which could have generated some more uncertainty. In addition, loops of bungee act different than normal bungee cord, so the two loops added uncertainty. Also the bungees may not have been adequately stretched in the beginning, so the values of the bungee may be tighter during the
first trials, and looser later on when the bungee had been used more. To reduce uncertainty, we could have had a more precise measuring system rather than using our eyes to see where the bottom of the bungee loop seems to lie.

**Conclusion:**

Because the trend lines from our Force over Displacement graph were all linear with a similar nonzero y-intercept, we can conclude that our bungee does not follow Hooke's Law perfectly, however it does follow a real-life version of Hooke's Law. It follows a real-life version of Hooke's Law, because the trend lines are linear. The average force intercept at 0.345 Newton for zero displacement could represent the amount of force that must be applied before any displacement in the bungee occurs because it is not an ideal frictionless and weightless spring. Knowing this, we can still apply the Hooke's Law to the bungee cord, however, we must take the little extra force that is required before any change in bungee length will happen. Taking the conclusions about Hooke's Law for the bungee, using the K-values, there is a relatively linear relationship between the K-value and 1/Length of Cord. Knowing this, we can predict what the K-value will be 0.8027 times the inverse length of the cord for a much larger length of the cord. However, using the data in Figure 4, we observed that as the cord gets shorter, the graph may approach infinity, however, the actual bungee would have a maximum K-Value. In addition, as the cord gets longer and longer, the K Value eventually approach a minimum nonzero K-Value, or possibly snap. So even though we may be able to predict a longer length, there may be a nonzero minimum K-value that may not fit our graph in Figure 5.