Bungee Drop 1: Hooke’s Law & the K-value of our cord

**Intro:** The goal of our experiment was to derive an equation for the k-value (known as the spring constant in springs) of our cord based on the given length of the cord. This concept of a k-value is based on Hooke’s Law. Hooke’s law simply states that the force needed to compress or extend a spring is proportional to the distance it compresses or extends the spring or cord. Equation 1 is a derivation of Hooke’s law when the system is at equilibrium. We used equation 1 extensively throughout the experiment.

\[
\text{Equation 1 } \ k = \frac{mg}{x}
\]

m = mass
\(g = \text{acceleration due to gravity (9.81 m/s}^2\)
\(x = \text{stretch of cord}

**Method:** In this experiment, we derived a formula for the k-value of the cord based on the initial length of the cord. We did this by finding the different k-values of the cord while varying the initial length of the cord and keeping the mass constant. The first thing we did was attach a pole with a metal rod to the side of our lab desk. The pole had a metal bar stick out, parallel to the floor, that gave us a hook approximately 2 meters above the ground to fasten our cord to. Once our cord was fastened to the hook and hanging to the ground, we tied another loop in the cord approximately 10 cm below the top knot. We then measured and recorded the value and found it to be 9.5 cm. Then we attached a 150g mass to the loop, and recorded the new distance in between the top knot and the loop. We then removed the mass and untied the knot but left the cord hanging from the top hook. We then tied a new loop further down the cord, recorded the distance from the top knot, added the 150g mass, and recorded the new distance. We repeated this process for a total of 5 different loops varying in initial length from 9.5 cm to 78 cm. This gave us all of the information needed to find the individual k values for cord at different lengths.
Results: We found that the k-value is inversely proportional to the initial length of the cord. We did this by first finding the k-value of our cord at different lengths and then plotted this data. This revealed an interesting relationship between the k-value and the initial length of the cord.

\[
\begin{array}{cccccc}
 L_i (m) & L_f (m) & x (m) & K (N/m) & 1/L_i (1/m) \\
 0.095 & 0.187 & 0.092 & 16 & 10.52631579 \\
 0.203 & 0.404 & 0.201 & 7.323383085 & 4.926108374 \\
 0.373 & 0.751 & 0.378 & 3.894179894 & 2.680965147 \\
 0.548 & 1.091 & 0.543 & 2.710865562 & 1.824817518 \\
 0.78 & 1.55 & 0.77 & 1.911688312 & 1.282051282 \\
\end{array}
\]

This table includes all of our results. The first column \((L_i)\) is the initial length of the cord, the second column \((L_f)\) is the final length of cord after the 150g mass was added, and the third column is the difference between the \(L_f\) and \(L_i\) or \((x)\). Because the cord mass was not accelerating upwards or downwards, we knew the system was at equilibrium. This meant that the force of gravity was equal to the force of the cord. Because we know the force of gravity is equal to the acceleration due to gravity \((g)\) times the mass \((m)\) and the force of the spring is equal to our k value times \(x\), we could derive equation 1. We used this equation to find the value of k for the cord in all of our trial and this is recorded in the 4th column. The fifth column is simply 1 divided \(L_i\), which is what we used to linearize our graph. This last column will make more sense as you see what the rest of our data looks like plotted on a graph.

![Graph of k vs. Li](image.png)
This graph shows the relationship between the initial length of the cord \( (L_i) \) and the k-value of the cord. As soon as we plotted the data, we noticed how similar the curve of the graph looked to a power graph of power \((-1)\). This is what led us to creating the fifth column in our chart. Once we actually found the equation for the curve, we saw that the power of the curve was so close to \((-1)\), that we should be able to linearize the graph by finding \(1/L_i\). Our next graph is exactly that.

![Graph showing k vs. 1/Li](image)

This graph is the relationship between \(1/L_i\) and k. As we guessed, the graph is linear and the equation has a y-intercept of 0. The fact that it runs through the point \((0,0)\) is another very important factor. This means our cord reflects an ideal Hooke’s law cord because when the force is equal to 0, so does the initial length and then for every point after that, there is a direct correlation between the two.

**Discussion:** The final equation we found in our experiment is extremely useful and important because it can be used to find the k-value of the cord at any length. Unfortunately this is only true when our mass is 150g. This is because, assuming our system obeys Hooke’s law, as we add or subtract mass to the system the value of \(x\) will change as well, which means we will end up with a different k-value.

There were also definitely sources of error or uncertainty. The uncertainty of the slope in our linear equation was .009. This gave us a percent uncertainty of approximately .6%. Unfortunately we do not have an accepted value for an equation for k-value of our cord and therefore cannot find our percent error. This would have been very useful but since we don’t have it we have to accept our results and use this equation when we attempt the bungee drop on the final lab day. As far as the raw uncertainty in our experiment, the only tool we used was a tape measure, which had a raw uncertainty of .01 cm.

When it comes to intangible uncertainties associated with our experiment the first thing that comes to mind is human error. More specifically, my partner or I may have not measured the initial length \( (L_i) \) of the cord correctly. This is because it was difficult to measure the string before the mass was attached because it was not very taut, and if we
applied a pressure to it to make it taut, it would have stretched and the initial length would change. Measuring the final length was not always an easy task either. We had to consistently measure from the same part of the top knot and the bottom knot. Making a mistake during this part could make a difference of as much as .5 cm per measurement. Similarly, we had to be very careful to tie the knots the same way. If one was larger than the other, it could lead to another source of uncertainty, as it would affect both the initial and final length of the cord. Of course all of this is assuming that our cord obeys Hooke’s Law perfectly. In theory it should and both of our sets of data seem to support this. In reality though, it is difficult to determine whether or not it really obeys Hooke’s Law and this is definitely another source of uncertainty.

**Conclusion:** In this experiment we successfully found the relationship between the initial length of our cord and the k-value of our cord. This of course was only true for a system of 150g. If I were to continue with this experiment, I would evaluate more in depth the relationship between the mass and the k-value. This part is just as important as the relationship between the initial length and the k-value when we actually design our experiment.