How can the spring constant of a bungee cord be found?

**Introduction:**

The objective of this experiment was to collect data that would be useful when designing the final bungee jump lab. The researchers decided that finding the \( k \) constant of the bungee cord would be beneficial. The \( k \) constant is the stiffness of an elastic system. Thus, the researchers conducted an experiment that applied Hooke’s law to a system consisting of a length of the bungee cord, \( L \), and a mass on a hanger, \( m \). The variable \( \ddot{x} \) was used to represent the displacement from equilibrium of the hanger when a mass was placed on it. The variable \( \sum \vec{F} \) was used to represent the vector sum of all forces acting on the system. Using these variables, Hooke’s law could be used to measure the \( k \) constant of the bungee cord:

\[
\sum \vec{F} = -k \ddot{x} \quad \text{(Equation 1)}
\]

The independent variable for this experiment was the force due to gravity on the system. The force due to gravity can be determined using the mass, \( m \), and the acceleration due to gravity, \( g \):

\[
\sum \vec{F} = m \ddot{y} \quad \text{(Equation 2)}
\]

The dependent variable that was measured was \( \ddot{x} \). Three different lengths of the cord were used in order to compare the \( k \) constants measured.
Methods:

In order to gather the correct data for the experiment, the length of the cord was kept constant for each set of trials was kept constant. The force on the system was varied by changing the amount of mass on the hanger.

Set-up

Figure 1: Diagram of Experiment
This diagram provides the set-up for this experiment and the instruments that were used.

This experiment was performed in a lab classroom in the science building of Washington and Lee University. A clamp was attached to the edge of a flat table. A pole is placed upright into the clamp. The pole has another clamp at its top end. The length of the bungee cord is measured and the halfway point in the middle of the cord is tied to the top clamp of the pole. The mass hanger is hung at the bottom of the bungee cord. The mass hanger is now considered to be
at equilibrium. Only one bungee cord was used in this experiment. For the first set of trials, half of the length of the bungee cord was hung. For the second set of trials, a quarter of the length of the bungee cord was hung. For the third set of trials, an eighth of the length of the bungee cord was hung.

Procedure

First, the researchers set up the experiment as indicated above. The length from the top of the bungee cord to the top of the mass hanger was recorded with a tape measure. This measurement represents the equilibrium length. The first set of trials began with a mass of 10g being placed onto the hanger. The displacement of the hanger from equilibrium was then measured. This process of placing a mass on the hanger and recording the displacement from the equilibrium position was repeated in 10g increments with 50g being the final mass. The cord was then removed and a quarter of the cord was measured. The quarter length of the cord was hung. The process in the first set of trials was repeated for the second set of trials. The cord was then removed and an eighth of the cord was measured. The cord was hung. The process in the first two sets of trials were repeated for the third set of trials. For the first set of trials, the trials with 60g and 70g could not be measure because the mass hanger touched the ground.

Results:

In the end, the results suggested that distance from equilibrium and force were proportional. The $k$ constant of the cord was able to be calculated using Hooke’s Law (Equation 1).

Initial Recorded Data

Length of unstretched cord = $2.49 \pm 0.005$m
Length of half of the cord = $1.25 \pm 0.005$m
Equilibrium length of half cord = 1.59m±0.005m
Length of quarter cord = .623m±0.005m
Equilibrium length of quarter cord = .74m±0.005m
Length of eighth cord = .311m±0.005m
Equilibrium length of eighth cord = .459m±0.005m
Mass of hanger = 50g±0.1g

Figure 2: Table of Recorded Forces and Lengths
All recorded and calculated data for all three sets of trials.

<table>
<thead>
<tr>
<th>Force from masses(N±0.001N)</th>
<th>Stretched Length(m±0.005m)</th>
<th>Stretch from Equilibrium(m±0.005m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half cord</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.491</td>
<td>1.590</td>
<td>0.345</td>
</tr>
<tr>
<td>0.589</td>
<td>1.700</td>
<td>0.455</td>
</tr>
<tr>
<td>0.687</td>
<td>1.890</td>
<td>0.565</td>
</tr>
<tr>
<td>0.785</td>
<td>1.945</td>
<td>0.700</td>
</tr>
<tr>
<td>0.883</td>
<td>2.100</td>
<td>0.855</td>
</tr>
<tr>
<td>Quarter Cord</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.491</td>
<td>0.740</td>
<td>0.117</td>
</tr>
<tr>
<td>0.589</td>
<td>0.780</td>
<td>0.157</td>
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<tr>
<td>0.687</td>
<td>0.825</td>
<td>0.202</td>
</tr>
<tr>
<td>0.785</td>
<td>0.890</td>
<td>0.267</td>
</tr>
<tr>
<td>0.883</td>
<td>0.955</td>
<td>0.332</td>
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<tr>
<td>0.981</td>
<td>1.030</td>
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</tr>
<tr>
<td>1.079</td>
<td>1.110</td>
<td>0.487</td>
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<tr>
<td>One-Eighth Cord</td>
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<td></td>
</tr>
<tr>
<td>0.491</td>
<td>0.385</td>
<td>0.074</td>
</tr>
<tr>
<td>0.589</td>
<td>0.400</td>
<td>0.089</td>
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<tr>
<td>0.687</td>
<td>0.430</td>
<td>0.119</td>
</tr>
<tr>
<td>0.785</td>
<td>0.458</td>
<td>0.146</td>
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<tr>
<td>0.883</td>
<td>0.490</td>
<td>0.179</td>
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<tr>
<td>0.981</td>
<td>0.530</td>
<td>0.219</td>
</tr>
<tr>
<td>1.079</td>
<td>0.570</td>
<td>0.259</td>
</tr>
</tbody>
</table>

Figure 2 Description:
The researchers used Equation 2 in order to calculate the force due to gravity on the system. The stretched length was measured from the top of the pole to the top of the hanging mass using a tape measure. The stretch from the equilibrium was calculated by subtracting the equilibrium length from the stretch length. The uncertainty of ±0.005m for the lengths is due to the smallest measurement on the tape measure being 0.01m. The uncertainty of ±0.1N is due to the uncertainty of the each mass being about 0.1g.
The forces and displacements were graphed for all three lengths of bungee cord with trend lines. The lowest data point for each data set was removed in order to make the data sets more linear. The trend lines and equations were calculated using Excel. Hooke’s Law (Equation 1) can be applied to each of the equations of the trend lines by replacing the $x$ variables with $\vec{x}$ and the $y$ variables with $\vec{F}$. Thus, the $k$ constant for each length of the bungee cord is the slope of its trend line. The uncertainties for the trend line equations were found using Excel. The equations with their uncertainties are as follows:

Half Cord: $y = 0.73\pm0.04x + 0.27\pm0.03$

Quarter Cord: $y = 1.46\pm0.06x + 0.38\pm0.02$

Eighth Cord: $y = 2.9\pm0.1x + 0.35\pm0.02$
Therefore, the $k$ constants were found to be:

Half Cord: $k = 0.73 \pm 0.04 \text{ N*m}$

Quarter Cord: $k = 1.46 \pm 0.06 \text{ N*m}$

Eighth Cord: $k = 2.9 \pm 0.1 \text{ N*m}$

**Figure 4: Graph of $k$ Constants versus Inverse Length**

The $k$ constants of each of the three cords versus the inverse of their unstretched lengths

**$k$ Constants versus Inverse Length**

![Graph of k Constants versus Inverse Length](image)

**Figure 4 Description:**

Using Figure 3, the $k$ constants were taken and plotted against the inverse of their unstretched lengths which are found in the initial recorded data section. According to Hooke’s Law (Equation 1), the $k$ constant and the length are inversely proportional. This chart shows that the relationship between the inverse of length and the $k$ constants is linear. The uncertainty of the equation of the trend line is: $y = 0.962x \pm 0.006 + 0.004 \pm 0.001$

The slope of Figure 4 should theoretically be 1. Therefore the percent error of Figure 4 is:

$$\frac{1 - 0.962}{1} \times 100 = 3.8\%$$

**Discussion:**

In this experiment, Hooke’s Law (Equation 1) was used to find the $k$ constants for different lengths of a cord. These constants were found by taking the slopes from the graphs. The
theoretical \( k \) constant of the cord is not known. Thus, the percent error cannot be calculated for this experiment. However, the percent uncertainties can be found for each of the \( k \) constants. The percent uncertainties for each of the lengths of cord are: half cord 0.73±0.04 N*m 5\%, quarter cord 1.46±0.06 N*m 4\%, and the eighth cord 2.9±0.1 N*m 3\%. The percent uncertainties are small which suggests that the \( k \) constants found are precise. It is interesting to note that the percent uncertainties actually decrease as the length of the cord becomes shorter. This constant decrease could be due to the cords becoming more stable as they become shorter. For the longest cord, the researchers had to wait a few seconds after placing the mass onto the hanger for the hanger to settle at a point. For the shortest cord, the hanger settled at a point almost immediately. Therefore, the stiffness, its \( k \) constant, of the spring is inversely proportional to the percent uncertainty of its \( k \) constant.

Figure 4 was graphed in order see how closely the \( k \) constants collected followed Hooke’s Law. The relationship between the \( k \) constant and the length of the cord should be proportional. Figure 4 shows that they are almost directly proportional. In fact, a percent error was calculated for the slope of the relationship between the inverse length and the \( k \) constant since it theoretically should be 1. The percent error of Figure 4 is only 3.8\%. This percent error shows that the experiment followed Hooke’s Law closely and that the information attained will be valuable when designing the final bungee jump.

While the error in this experiment is low, there are still ways that it can be minimized. The friction that is present in the experiment is not accounted for in the measurements and the calculations. The bungee cord in itself is definitely not friction free. Thus, some energy is lost in each trial to the force of friction. To minimize the error, the force of friction can be estimated and then incorporated into the calculations.
Conclusion:

The $k$ constants of the bungee cord for three different lengths were successfully found by changing the force due to gravity of a system and measuring the displacement from the equilibrium. The percent error of the relationship between the $k$ constants and the length of the cords was 3.8% which shows that the experiment was successful in using Hooke’s Law. The researchers will use the $k$ constants found in order to create their design for the bungee jump. The $k$ constant will be useful when calculating elastic potential energy. This is because the equation for elastic potential energy is $PE = \frac{1}{2} kx^2$. 