Is Hooke's Law Observed for an Elastic Bungee Cord?

Introduction:

Bungee jumping is an extreme activity for those who enjoy the sense of danger when they are in a controlled situation. A jumper can jump off of a tall platform or bridge and enjoy the feeling of falling until he or she reaches the end of the bungee cord. The bungee cord then applies a force on the jumper to slow him or her, and finally, to bring the jumper to rest. The length of the bungee is crucial for a successful bungee jump. Too short of a bungee cord will result in a poor experience for the jumper and too long a bungee cord may end in serious injury or death.

To figure out exactly how long a bungee cord should be so that the bungee jump is an ideal experience for the jumper, it must first be tested to find out its degree of stretchiness. In this particular experiment, the “bungee cord” being tested is a thin rubber strand that will be used for an egg to make a “bungee jump” from the fourth floor balcony of Washington and Lee University’s Great Hall, to the second floor of the Great Hall.

If the bungee cord acts as a spring when it stretches, it will obey Hooke’s Law, as shown in Equation 1 where \( F \) is the magnitude of the force on the spring, \( x \) is the magnitude of the displacement of the spring from its equilibrium (resting) position and \( k \) is the spring constant (i.e. the measure of the spring’s stretchiness.) Since the force on the bungee will only be caused by the acceleration of gravity on the mass that is hanging on it, the force on the bungee will also equal the mass on the bungee, \( m \) multiplied by the acceleration of gravity, \( g \).

\[
F = k \cdot x = m \cdot g \tag{1}
\]

If this is the case, then a linear relationship between a force on the end of a bungee cord and the amount that the cord is stretched should be observed. However, if a more complicated trend exists, then a nonlinear relationship between the force on the bungee cord and the displacement of the bungee cord will be observed, indicating that a single \( k \) value cannot be used to predict the distance that the bungee cord will stretch when a given force is applied to it.

Since the force on the bungee cord is created by a hanging mass on the bungee cord and the acceleration of gravity, the mass of the object can be used to calculate a \( k \) value for the given situation because the acceleration of gravity can be absorbed to the \( k \) value to create a new \( k \) value \( (k_1) \) as shown in Equation 2.

\[
m = \frac{k}{g} \cdot x = k_1 \cdot x \tag{2}
\]
Methods:

In order to test the relationship between the force placed on a bungee cord and the displacement of the bungee cord, a small, hanging bungee cord setup was created. This setup was used with the intention that the mass on the cord could easily be varied, the displacement of the cord could be easily measured, the length of the cord could be easily changed, and the number of stands that created the bungee cord could be easily manipulated.

To begin, a tape measure was hung from a metal beam that had three screws that could be used as hooks running through it. The beam was clamped to a pole so that the pole was perpendicular to the ground and the beam was parallel to the ground. The pole was vertically clamped to a nearby table so that it had more height (Figure 1). The bungee cord was taken out of its initial packaging and stretched in portions of about one foot until the entire length of the bungee cord had been stretched. This was repeated a total of three times so that the un-stretching of the cord would not occur over the course of the experiment. The bungee cord was tied in a manner that is much like one would use to tie off a balloon so that the knots could easily be taken out, and so that a small loop was formed.

For the first set of trials, two knots were tied about 0.40m apart, with one of the knots close to the end of the bungee cord. The knot that was farthest from the end of the bungee cord was hung from screws/ hooks on the metal beam, and the excess cord was wrapped around the pole so that it was out of the way. The length from the start of the tape measure to the top of the hanging knot was measured, and then 0.018m was subtracted from this length, as this was the distance between where the measuring tape started and the point where the bungee cord had been hung. A mass of 0.050kg was then hung from the hanging loop, and the total length of the displaced cord was found. The mass was then removed until the next trial to prevent stretching. The amount of mass hung on the hanging loop was then increased by 0.020kg for a total of 5 trials and then increased by 10kg for two more trials. This procedure was later repeated for a bungee cord with a resting length of about 1.000m and then again for a bungee cord with a resting length of 1.500m.

Another trial was then performed where a bungee cord with an initial length of about 0.40m was measured, followed by a measurement of the total length of the cord when a 0.050kg mass was hung from it. The length of the cord was then increased by about 0.100m and the total displaced caused by the 0.050kg mass was again measured. This procedure was then repeated a total of eight times.

The very first procedure that was described was then repeated using two strands of bungee cord with a resting length of about 0.040m, instead of just a single strand. Nothing else was varied from the first set of procedures. The data from all of the trials and setups was then graphed and best-fit equations were calculated. $R^2$ values of greater than 0.97 were used to determine the best-fit equations because 0.97 appeared to be the $R^2$ value at which the trend lines passed through all of the data points. Once an equation was found with an $R^2$ value of greater than 0.97, the search for a best-fit equation was stopped because of the small number of data points present, which meant that high degree polynomials could be found that would create an $R^2$ value of 1.00 but would not actually reflect the general trend in data.
Results:

The trials involving the amount of stretch caused by placing different masses on a bungee cord of constant length were graphed so that the length stretched vs mass hung on the bungee cord ($x$ vs $m$) could be observed. The relationship between $x$ and $m$ seemed to be non-linear and different for each of the trials. The correlation between the resting length of the cord and the length when a constant mass was added to the cord ($x$ vs $x_0$) however, did exhibit a linear relationship with a constant of 1.173. The trial involving two strands of bungee cord also appeared to exhibit a linear relationship between mass added and the stretched length of the bungee with a slope value of 0.736.

The relationship between the three different trials that observed the trends between the total mass that was added and the stretched length of the bungee cord ($x$ vs $m$) all appeared to have different quadratic relationships (Graph 1). The uncertainty in the mass was ± 0.001kg since the masses used were pre-weighted mass standards. The uncertainty in the length was ± 0.001m and came from uncertainty in the tape measure used.

Figure 1: Bungee Cord Setup

The bungee cord was knotted in two places and then hung from the hooks on a beam that was clamped to a table so that its displacement due to the added mass could be found.
For the experimental setup where the difference between the resting length and the stretched length of the bungee cord \((x \text{ vs } x_0)\) was observed when a 0.050kg mass was hung from the bungee, the relationship between the two factors was found to be linear with a slope of 1.173 (Graph 2). The uncertainty for both the stretched and resting values was ± 0.001m and came from uncertainty in the tape measure.

**Graph 1: Best-Fit Relationships of Displacement vs Mass.** The three different resting lengths of bungee cord; the displacement for each trial was plotted against amount of mass that was hung, and the best-fit relationship was determined.
In the experimental setup where two strands of bungee cord were used together, the various stretched lengths of the bungee were graphed against the masses used to produce them (Graph 3). The relationship between the two factors was determined to be linear with a slope of 0.736 and an $R^2$ value of 0.989. The uncertainty in the mass was ± 0.001 kg since the masses used were pre-weighted mass standards. The uncertainty in the displacement was ± 0.001 m and came from uncertainty in the tape measure used.

**Graph 2: Resting Length of the Bungee Cord vs Stretched Length of the Bungee Cord.** The resting lengths of the resting bungee cord were plotted against the length that the bungee cord was stretched by a 0.050 kg mass.
The ratio between the resting length and the stretched length of the single stranded bungee cord was then created for masses other than 0.050 kg using parts of the data that had been obtained from the three experimental setups that measured the change in stretched length of the bungee cord as the mass on it was increased (Table 1). This ratio was considered to be equivalent to the slope found in Graph 2 for the relationship between resting length and stretched length of the bungee with the use of a 0.050 kg mass. The ratios were then graphed against the mass used, and the best-fit relationship between them was determined (Graph 4).

**Table 1: Mass and Ratio.** The ratios between the resting length and stretched length of the bungee cord when different masses are used.

<table>
<thead>
<tr>
<th>Mass $m$ (kg) ± 0.001 kg</th>
<th>Ratio $(x/x_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>1.175</td>
</tr>
<tr>
<td>0.070</td>
<td>1.220</td>
</tr>
<tr>
<td>0.090</td>
<td>1.306</td>
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<tr>
<td>0.110</td>
<td>1.424</td>
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<tr>
<td>0.130</td>
<td>1.578</td>
</tr>
<tr>
<td>0.150</td>
<td>1.737</td>
</tr>
</tbody>
</table>
Discussion:

The graph of the relationship between the three experimental scenarios where resting length of a single stranded bungee cord was kept constant while varying masses were added to the bungee cord suggests that Hooke’s Law is not obeyed by the bungee cord (Graph 1). If it were, all three of the experimental setups should exhibit straight lines with identical slopes. Instead, each of the scenarios exhibits a very different quadratic function as its best fit equation.

The scenario where the relationship between the resting length of the bungee cord and the stretched length of the bungee cord when a 0.050kg mass was added to it was found to be linear. This suggests that the relationship between resting length and stretched length might be linked to a useful way to determine the relationship between the mass on the bungee cord and the displaced length of the bungee cord. To this end, Table 1 was created after the experiment to see if any sort of relationship between the ratios and the different masses could be found. The relationship between these ratios and their masses was then determined by finding the best-fit equation, which was:

\[
\frac{x}{x_0} = 37.589m^2 - 1.801m + 1.167
\]

This means that if the mass that is placed on the bungee cord is known and the distance that the bungee cord should stretch is known, then the resting length of the bungee cord that one should use can be calculated. However, it is likely that this equation will break down when it is used with masses that are heavier than the ones that have been tested, because the relationship between displaced length of the bungee and mass used on the bungee cannot truly be quadratic. The three very different quadratic equations found in the three setups that examined the
relationship between mass added and the displaced length of the bungee cord demonstrate this, as they would all be identical if the relationship was truly quadratic. If larger masses were to be used on the bungee cord, it would be advisable to do more testing on the bungee cord with those masses to determine the ratio between $x$ and $x_0$ from empirical data.

While the double stranded bungee cord appeared to have a linear, relationship based on the $R^2$ value, it is unlikely that this would be observed if heavier masses had been used, as the single stranded counterpart of this experiment showed that the relationship between mass used and total length stretched was found to be quadratic. It is likely that the stiffening of the bungee cord caused by making it two strands thick minimized this effect with the masses used, and that further experimentation with heavier masses that created larger amounts of stretch on the bungee cord would likely cause a more quadratic relationship.

Uncertainty in this lab was fairly minimal, as pre-weighted standard masses and accurate tape measures were used. However, error in this lab was abundant. Every time that the bungee cord was stretched, it recoiled to only a fraction of the amount that it had been stretched. While this stretching was assumed to be fairly minimal, and ignored, it nonetheless had an effect on the data by causing the bungee cord to be slightly less stretchy as the experiment progressed. Other sources of error in this experiment came from the differences in the placement of the knots on the bungee cords. Although the knot near the end of the bungee cord was relatively close to the end of the bungee cord, the small length of cord between the end of the bungee cord and the placement of the knot added a small amount of mass to the system. Since the knot had to be retied several times for changes in length and the changes in the number of strands, this meant that the amount of mass added by the length of cord below the knot changed slightly each time that the knot was retied. This small change in mass was not taken into account and thus the actual mass on the cord could have varied slightly from trial to trial. The size of the knot also changed slightly each time the knot was retied, which meant that the amount that the knot contributed to the stretching of the bungee cord also changed slightly for each trial where the knot was retied. Attempting to tie the smallest knots possible minimized this effect, as it help to ensure that all of the knots were approximately equal in size.

**Conclusion:**

The bungee cord does not obey Hooke’s Law, so a more complicated way to predict the stretched length of a bungee cord is needed. The linear relationship between the resting length of a bungee cord and its stretched length when a given mass is hung from the bungee is a possible way to calculate the relationship between a given mass, resting length and stretched of the bungee cord. Equation 3 suggests a relationship for these ratios but further experimentation is needed. Thus, future experiments could look at the relationship between mass, resting length and stretched length at masses that are greater that 0.150kg. This could allow a more solid relationship between the mass, resting length and stretched length of the bungee cord to be found, which would allow the ideal length for a bungee cord to be found for the egg bungee jump in Washington and Lee’s Great Hall.