Modeling the Relationship between Bungee Cord Length and Displacement

1. Introduction

This experiment is the second in a pair of experiments leading up to the Bungee Challenge, which involves dropping an egg attached to a bungee cord in the Great Hall. The goal is to get the egg as close to the ground as possible without breaking the egg and to have the egg experience the greatest acceleration below 3g.

The purpose of this experiment is to determine the relationship between the length and the dynamic displacement of a bungee cord using different cord lengths and masses. This relationship will be based on the ideal spring model, Hooke’s Law, which states

\[ F = -kx \]  

where \( F \) is the restoring force of the spring, \( x \) is the displacement from the equilibrium point, and \( k \) is the spring constant. The goal of this experiment is to determine the relationship between \( k \) and the length of the bungee cord \( L \), altering Hooke’s Law to apply it to the bungee cord, which is not an ideal spring.

2. Methods

The experiment involved measuring the displacement of a bungee cord when a hanging mass attached to the end of the cord was dropped from the top of the bar to which the cord was attached. The displacement of the cord was measured for seven different weights, and five lengths of the bungee cord were tested.

**Figure 1: Experimental Set-up.** The pre-experimental set-up is shown on the left, along with the set-up during the displacement measurement on the right.
The experiment was set up on a table in the lab room, using a bar attached to the table by a clamp. The tape measure was attached to the top of the bar using tape, and it was also taped to the floor to prevent it from moving. A loop was tied at the bottom of the bungee cord, and a separate loop was made higher up the cord and tied around one of the washers on the bar. This created a bungee cord with length $L$ from the top of the bar to the bottom of the loop on the end, as shown in the diagram on the left in Figure 1. The excess cord was secured behind the other washers in order to prevent interference. The set-up also included a 50g hanging mass and additional masses ranging from 10g to 100g. An iPad was used to capture the video of the drop using the CoachMyVideo app.

Once the experiment was set up, we began to measure the displacement of the cord. Starting with an unstretched cord length of 0.13m, we placed a 100g total hanging mass on the bottom loop of the cord. One partner lifted the hanging mass to the point where the top of the hook of the hanging mass was level with the top of the bar. The partner dropped the mass while the second partner watched to see how far it dropped. Once the approximate maximum displacement point was determined, the second partner prepared to record the drop with the iPad in the approximate location of the maximum displacement point, making sure to include the measuring tape in the frame. The mass was dropped again, with the second partner recording the drop on the iPad. We played the video back to find the point where the hanging mass changed direction, and we recorded the distance reached by the bottom of the hanging mass. This process was repeated for total hanging masses of 120g, 140g, 150g, 160g, 170g, and 200g. Once the displacement was recorded for each mass, we repeated the steps for lengths of 0.153m, 0.217m, 0.234m, and 0.267m.

3. Results

The results of the experiment indicate there is a linear relationship between the force $F$ and the displacement $x$ for each length of the cord. The slope of this relationship, $k$, varied for each length $L$, but there was also an inverse relationship between $k$ and $L$.

**Figure 2:** Bungee Cord Displacement. The displacement $x$ measured using various weights with different cord lengths.

<table>
<thead>
<tr>
<th>Mass $m$ (kg, ± 0.01kg)</th>
<th>Force $F$ (N, ± 0.01N)</th>
<th>Displacement $x$ (m, ± 0.01m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L = 0.130m$</td>
<td>$L = 0.153m$</td>
</tr>
<tr>
<td>0.10</td>
<td>0.98</td>
<td>0.41</td>
</tr>
<tr>
<td>0.12</td>
<td>1.18</td>
<td>0.46</td>
</tr>
<tr>
<td>0.14</td>
<td>1.37</td>
<td>0.51</td>
</tr>
<tr>
<td>0.15</td>
<td>1.47</td>
<td>0.53</td>
</tr>
<tr>
<td>0.16</td>
<td>1.57</td>
<td>0.54</td>
</tr>
<tr>
<td>0.17</td>
<td>1.67</td>
<td>0.58</td>
</tr>
<tr>
<td>0.20</td>
<td>1.96</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Figure 2 shows each hanging mass $m$ in kg, the force $F$ of the weight of each mass ($m \times 9.81 \text{ m/s}^2$), and the displacement $x$ at each of the five cord lengths $L$. There is a clear trend in the data, showing not only an increase in $x$ as $m$ increases for each constant $L$, but also an increase in $x$ as $L$ increases for each constant $m$. 
**Figure 3: Force vs. Displacement.** The slope of each cord length’s force versus displacement line is the $k$-value for that length.

![Force vs. Displacement](image)

Figure 3 shows the relationship between the force $F$ of the weight and the displacement $x$ at each cord length $L$. There is a linear relationship for each length, the slope of which is the Hooke’s Law $k$-value for that particular length. As the length of the cord increases, the slope of the force versus displacement line decreases, indicating an inverse relationship between $k$ and $L$.

**Figure 4: k-value vs. Inverse Cord Length.** The graph shows a linear relationship between $k$ and $1/L$. 

![k-value vs. 1/Length](image)
Figure 4 shows the linearized relationship between $k$ and $1/L$, represented by the equation $k = 0.5801(1/L) - 0.3766$. The slope of the relationship is $0.5801 \pm 0.0266$, including the standard error which was determined. The y-intercept of the line is $-0.3766 \pm 0.1484$. This equation is used to find the spring constant equivalent for a certain length of string, and that spring constant is used in Equation 1 to determine the cord’s displacement due to a certain force. The modified version of Hooke’s Law can be given as $F = -(0.5801(1/L) - 0.3766)x$.

4. Discussion

The results of the experiment indicate the bungee cord follows a modified version of Hooke’s Law, with the spring constant varying based on a linear relationship with the inverse of the cord’s length. Equation 1 works for each length of the bungee cord, but the $k$-value is not constant for the different lengths, unlike the $k$-value for an ideal spring.

The force versus displacement lines in Figure 3, which represent Equation 1 for the various lengths, all have y-intercepts. Theoretically, however, their y-intercepts should be 0 since Equation 1 does not have an intercept. Their intercepts are due to experimental uncertainty, of which there were several sources in the experiment.

The first source of error was the method of measuring the displacement. While the iPad video was able to be viewed frame by frame, it was uncertain whether the frame stopped exactly at the maximum displacement point, meaning the measurements could have been slightly off. The loops in the bungee cord were another source of uncertainty because the cord was doubled at the loops, so the stretch property of the cord at those points wasn’t the same as a rest of the cord. Another source of error was the change in the loops after dropping the hanging mass multiple times. Sometimes, the force of the weight pulling on the top loop tightened it, increasing the un-stretched length of the cord and therefore affecting the measured displacement.

Future considerations for repeating this experiment could include dropping a weight several times before measuring the initial un-stretched length of the cord, allowing any tightening of the loops to occur before the cord is measured. Due to time constraints, only one displacement measurement was able to be made for each weight, so another change for future experimentation could be to do multiple trials for each weight, giving an average displacement rather than relying on just one measurement.

Another consideration to be made is the way in which the displacement was measured. For this experiment, the maximum displacement was measured at the bottom of the hanging mass, but since the displacement should be the change in length of the cord, the experiment’s displacement measurements are actually greater than they should be. This means the equation modeling the relationship between $k$ and $1/L$ is not correct. To fix this, the length of the hanging mass will need to be measured and subtracted from every displacement measurement. The results will then be recalculated using the new data, providing a new final equation for the relationship between $k$ and $1/L$.

The next step for this experiment would be to recalculate the displacement excluding the additional length of the hanging mass and change the resulting equations accordingly. The equations could also be improved by using more weights and lengths of the cord, in addition to doing more trials for each weight and length combination.
5. Conclusion

The purpose of the experiment was to model the dynamic behavior of a bungee cord using Hooke’s Law as a theoretical basis. The results indicated Equation 1 was followed for each length $L$ of the cord, but that there was a different $k$-value depending on $L$. We were able to conclude that there is a linear relationship between $k$ and $1/L$.

Although the equation found in Figure 4 is incorrect for this bungee cord due to the displacement measurement method, the results support the hypothesis that the bungee cord can be modeled using a modified version of Hooke’s Law. Recalculating the results will confirm the hypothesis and provide the correct equation for this bungee cord, which will allow us to determine which length of the bungee cord we need for the bungee challenge.