Characterizing a Cord: Studying the Effect of Varying Cord Length on the Displacement of a Bungee Cord

Introduction

One of the traditional thrill-seeker recreational sports is bungee jumping. Bungee jumping requires a person’s trust in the equipment that he/she is using. The appropriate length of bungee cord for a jump is the difference between thrilling and perilous. In this study, we aimed to characterize the effect of cord length on the stretch of a bungee cord, so that we can determine a safe length of cord for future jumps. The responses of an elastic cord can sometimes be compared to that of a linear spring. In this experiment, in order to look at the relationship between force and displacement, we used Hooke’s Law:

\[ F_{spring} = -kx\hat{i} \]  

where \( F_{spring} \) is the force the spring exerts on the mass (N) although here we are relating the spring to the bungee cord, \( k \) is the spring constant (N/m), \( x \) is the displacement (m), and \( \hat{i} \) is the direction of the vector.

Understanding this relationship between force and displacement helped us characterize the cord, specifically looking at \( k \). A spring constant, here applied to the bungee cord, gave us an idea of how much force (N) is required to stretch the cord by one meter. Ultimately, we found that an increase in the length of the bungee cord used resulted in a decrease in \( k \), since more rope led to greater displacement.
Methods

In this study, we hung various lengths of bungee cord on a stand and measured the displacement at each length in response to various hanging masses. Then, we used this data to generate graphs to demonstrate the relationship between force and displacement and to get an idea of the constant k for our specific bungee cord.

Set up: Our materials included a single bungee cord, tape, various masses, a hanging weight holder, a tape measure, and a stand. The bungee cord was taped to the top of the stand (Figure 1). A loop was tied at the bottom of the cord in a way that minimized the distortion of the cord as weights were hung from the loop. Next to the cord, our tape measure also hung down next to the bungee cord.

Figure 1. Experimental set up with free body diagram.
Procedure: First, we tied a knot in the end of our bungee cord to create a loop from which to hang the masses. Next, after determining approximately the length of cord we wanted to test, we secured the cord to the stand with tape, and then secured the measuring tape on the stand with tape as well. We then measured the actual length of the cord that we utilized for that trial. Following, we hung eight different masses ranging from 0.10 kg to 0.17 kg in 0.01 kg intervals on the cord, measuring the displacement of the cord (m) for each weight. We repeated this process for four other cord lengths. We graphed displacement against weight for each of the lengths and generated a line of best fit for the five graphs. Finally, we plotted the slope of each line of best fit against the length of the cord used, and then generated a line of best fit for this graph. This allowed us to see the trend of the k constant for our cord.

Results

Plotting displacement against weight for the first cord length of 0.198 m generated a line of best fit with a slope of 0.153 (Figure 2). For l = 0.475 m, displacement against weight resulted in a line of best fit where the slope equaled 0.365 (Figure 3). Furthermore, when l = 0.572 m, the slope of the best fit line for the graph of displacement against weight was 0.464 (Figure 4). When l = 0.690 m, slope equaled 0.556 for the slope of the best fit line for displacement against weight (Figure 5). The last cord length tested was 0.932 m, where the graph resulted in a slope of 0.738 (Figure 6).
Figure 2. Displacement (m) against weight (N). This graph shows the displacement of the bungee cord against the weight of the hanging masses when the length of the bungee cord is equal to 0.198 m.

Displacement vs. Weight for l = 0.198 m

\[ y = 0.1531x - 0.076 \]
\[ R^2 = 0.99202 \]

Displacement vs. Weight for l = 0.475 m

\[ y = 0.365x - 0.1736 \]
\[ R^2 = 0.9874 \]
Figure 3. Displacement (m) against weight (N). This graph reflects the displacement against weight of mass for the length of the bungee cord of 0.475 m.

\[
y = 0.4639x - 0.2426 \\
R^2 = 0.99515
\]

Figure 4. Displacement (m) against weight (N). The graph of displacement against weight for when length is equal to 0.572 m.

\[
y = 0.5561x - 0.2837 \\
R^2 = 0.99783
\]

Displacement vs. Weight for \( l = 0.572 \) m

Displacement vs. Weight for \( l = 0.690 \) m
Figure 5. Displacement (m) against weight (N). Displacement against weight for the cord length of 0.690 m is represented here.

Figure 6. Displacement (m) against weight (N). For the longest length of 0.932 m, this is the graph of displacement against weight.

Additionally, we then took the slope for each line and the length of each cord and plotted slope against length to generate a line of best fit for that graph (Figure 7). The slope of this graph was 0.805. We reasoned the uncertainty of the length of the cord and of the displacement measurements to be ±0.002. Using the following equation:

\[ U = (U_1^2 + U_2^2)^{1/2} \]  

where \( U \) is the overall uncertainty, \( U_1 \) is the least count for the tape measure (0.001 m), and \( U_2 \) is the additional uncertainty of our eyesight comparisons of the bottom of the knot to the measure tape, equal to the width of the knot in the cord (0.002 m).
Discussion

Using this static experiment, we hoped to better understand the behavior of our bungee cord. The final graph of the slopes of the lines of best fit plotted against the length of bungee cord used resulted in a positive slope. This pattern shows that slope increased as the length of the cord increased. In other words, the displacement of the cord was proportionally larger as the length of the cord grew. Given our $R^2$ values sat between 0.987 and 0.998, we have confidence in our observed trends. We see that a longer bungee cord with a smaller $k$ matches the behavior of a spring with a smaller spring constant, as predicted by Hooke’s Law (1). Possible sources of uncertainty include having to make a visual comparison between the location of the knot and the tick marks on the tape measure, which gives us an uncertainty of $\pm 0.002$. In the future, we would try to minimize uncertainty further by reducing the
size of the knot and bringing the tape measure as close to the cord as possible while both are still straight.

Conclusion

Recall that Hooke’s Law (1) allows us to relate force, displacement, and the nature of the bungee cord, expressed as constant k. With longer lengths of cord, displacement increases, independent of mass (since the same range of masses were used across the different lengths). Given this, we can conclude that the value of k, the amount of force it takes to stretch our bungee cord one meter, decreases as length increases. This is an important response to keep in mind when determining what length of bungee cord to use in a jump; as the height of the jump increases, the longer the cord used, the more displacement that occurs, so that smaller changes in the bungee cord would be appropriate for larger changes in jump height.

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