Relationship between mass, height of drop, and bungee cord length in a bungee drop

I. Introduction

In the Bungee Challenge project, we are to design a bungee cord that, when dropped with a certain mass (an egg) attached, will be able to get as close to the ground as possible with low maximum deceleration. Because the mass of the egg and the height of the drop will be measured and known, it is important to find a proper length of the bungee cord in relation to the known variables to perform a successful drop. As the behavior of the dropped mass from a certain height is similar to a mass object in free-fall, the potential energy due to gravity equation of the mass may be useful in our analysis.

\[ \text{Equation 1: } PE_{\text{gravity}} = mgh \]

Where \( m \) is the mass of the object, \( g \) the local gravitational constant, and \( h \) the height of the drop. Since \( m \), \( g \), and \( h \) are all values that will be known when the mass is dropped, \( PE_{\text{gravity}} \) will be known. In order to find an appropriate length of the bungee cord, we related the potential energy due to gravity of the object to the potential energy of the bungee cord to that of a spring after it has been displaced by the mass and set them equal to each other.

\[ \text{Equation 2: } PE_{\text{spring}} = \frac{1}{2}kx^2 = mgh \]

Where \( k \) is the spring constant of the bungee cord and \( x \) is the displacement of the bungee cord from equilibrium when mass is dropped, assuming the bungee is an ideal spring. The displacement is found by subtracting the height reached by the drop by the length of the bungee cord with no mass hanging from it.

In this experiment, we sought to analyze the behavior of the bungee cord at different lengths with varying masses attached and the height the mass would reach at each variation. Modeling this behavior will help us predict the appropriate design for the final egg bungee jump.

II. Methods

In order to find a relationship between mass, height traveled, and length of bungee cord, we chose to vary mass at a given length of the bungee to measure the total height the mass travels on the bungee. This will help us determine if there is a linear relationship between
height and mass at a given cord length. To find this, we measured 4 different cord lengths, ranging from 0.200m to 0.485m. For each cord length we tested five masses (0.050kg, 0.080kg, 0.090kg, 0.10kg, 0.11kg), with the exception of the longest cord length we measured (0.485 m) where the last two masses were smaller to prevent the hanger from hitting the ground. The total height reached by each mass on each length of bungee cord was measured from the top of the hanger where a knot is tied for the bungee cord at the top of the drop to the bottom of the hanger at the maximum height of the drop. Then the total height of the drop is also subtracted from the measured value because the starting point of the drop of the hanger has a difference of the height of the hanger from the actual height of the drop. We did two trials for every mass at a given cord length and took an average of the two trials to get a final height value.

![Figure 1: Setup. Where 1. is the length of the cord before attaching a mass to it, 2. the hanger at the initial position of the drop, and 3. the hanger at the maximum point of the drop (height of the hanger is subtracted from the measured height to give height of the drop).](https://via.placeholder.com/150)

**III. Results**

From the methods described above, we collected data from the measurements of the height of drop for each cord length with different masses. In doing so, we found the relationship between mass, cord length, and average height of drop to be linear. This information may be able to help us predict appropriate lengths for our bungee cord for the egg bungee challenge.
Table 1. Average height of drop (h) vs. mass at different bungee cord lengths – height of drop measured with varying cord lengths and varying masses for every cord length. Height of drop and cord length both have an uncertainty of ±0.01m.

<table>
<thead>
<tr>
<th>cord length, L (m)</th>
<th>0.200</th>
<th>0.295</th>
<th>0.440</th>
<th>0.485</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (kg)</td>
<td>ave h (m)</td>
<td>ave h (m)</td>
<td>ave h (m)</td>
<td>ave h (m)</td>
</tr>
<tr>
<td>0.05</td>
<td>0.414</td>
<td>0.624</td>
<td>0.9515</td>
<td>1.049</td>
</tr>
<tr>
<td>0.08</td>
<td>0.5515</td>
<td>0.824</td>
<td>0.1259</td>
<td>1.4015</td>
</tr>
<tr>
<td>0.09</td>
<td>0.584</td>
<td>0.895</td>
<td>0.1345</td>
<td>1.5265</td>
</tr>
<tr>
<td>0.1</td>
<td>0.609</td>
<td>0.9765</td>
<td>0.1469</td>
<td>—</td>
</tr>
<tr>
<td>0.011</td>
<td>0.704</td>
<td>0.1029</td>
<td>0.15825</td>
<td>—</td>
</tr>
<tr>
<td>0.06</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.159</td>
</tr>
<tr>
<td>0.07</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.259</td>
</tr>
</tbody>
</table>

Figure 2: Mass vs. height of drop. Average height values for different masses for four different cord lengths.

From the above graph, the linear trendlines for all four cord lengths between mass and height of drop seem to have linear relationships. That is, as as more mass is added, the height of the drop directly increases with it. However, it is hard to tell whether the variation in the length of the cord is also directly correlated to the height of the drop. Therefore, the data was manipulated and graphed below to determine the relationship between cord length and height of drop.
Figure 3. Cord length vs. average height of drop. Average height values for four different cord lengths at 5 different masses.

Here, the cord length, instead of the mass, was set as the independent variable on the x-axis and height of drop as the dependent variable on the y-axis. From the graph, there appears to be a linear correlation, where as cord length increases, height of the drop increases linearly. Masses 0.06 kg and 0.07 kg were left out because only one data point was measured for each mass.

Even though the two graphs above show a positive linear relationship between mass and height of drop as well as cord length and height of drop, they cannot give a comprehensive prediction of cord length based on varying height of drop and mass patterns because there are multiple linear trend lines associated with each graph. Therefore, the data was manipulated once more to give a more comprehensive look at the relationships of height of drop, cord length, and mass.

Table 2. Height per length ratio for varying mass— averaged height/length values for 5 different masses.

<table>
<thead>
<tr>
<th>mass (kg)</th>
<th>h/0.20m</th>
<th>h/0.295 m</th>
<th>h/0.44 m</th>
<th>h/0.485 m</th>
<th>ave. h/l</th>
<th>StdDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.07</td>
<td>2.115</td>
<td>2.163</td>
<td>2.163</td>
<td>2.128</td>
<td>0.02</td>
</tr>
<tr>
<td>0.08</td>
<td>2.758</td>
<td>2.793</td>
<td>2.861</td>
<td>2.89</td>
<td>2.825</td>
<td>0.04</td>
</tr>
<tr>
<td>0.09</td>
<td>2.92</td>
<td>3.034</td>
<td>3.078</td>
<td>3.147</td>
<td>3.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>
In the above table, height/length values for each mass were calculated and then averaged to produce one value, a constant. Two height values for the 0.485 m cord were left out for accuracy because they both had different mass values and there was not enough height measurements for the masses added.

**Figure 4. Mass vs. height/length**

![Mass vs. height/length graph]

In the graph above, there appears to be a linear relationship between height of drop divided by cord length and mass can be seen through the trend line:

\[ y = 22.954x + 0.9776 \]

Where x is the mass hanging from the bungee cord and y is the height/length. From equation 3, we can see that as mass increases, height/length directly increases with it. Using linear regression, the standard error of the slope was found to be 0.6 m, or 3% uncertainty, and the standard error of the y-intercept was found to be 0.06 m.

**IV. Discussion**

The purpose of our experiment was to determine the relationships between mass of an object, height of drop, and cord length for a bungee cord. In doing so, we discovered a
seemingly linear relationship between mass and height of drop, height of drop and length of the bungee, as well as mass and the ratio of height to bungee length.

Initially, we began with CWE theorem equations that described the behavior of an ideal spring when dropped with a mass attached, assuming the bungee cord behaved as an ideal spring. In equation 2, we can manipulate the $\text{PE}_{\text{spring}}$ equation so that it is in terms of the height of drop and the length of the bungee:

$$Equation 4: \frac{1}{2} k(h-L)^2 = mgh$$

Through this substitution of variables we can see that $(h-L)^2$ is proportional to $mh$, but direct relationships between the variables are difficult to quantify and model. Therefore, to find the direct relationships between the variables, we simply manipulated the data we took from measuring the height of the drop for different masses at varying cord lengths and found the ratio of height of drop to length of cord and the mass. Manipulating the data in this way helps to compile data together and simplify the relationships between the three different variables. Since there seems to be a linear correlation between $h/L$ and mass, as shown by equation 3, we can use this equation to help us predict the length of the bungee cord to use for the egg bungee drop because the mass of the egg and the height of the drop will be known, without having to take into account the k-constant or the force of gravity.

In the last bungee experiment, we also found that our bungee cord does not obey Hooke’s law, and is therefore not an ideal spring. Instead, it obeys a modified version of Hooke’s law. Because of this, the graphs for this experiment all have y-intercepts, which is not characteristic of ideal springs that obey Hooke’s law. The y-intercept for this experiment in equation 3 has a standard error of 0.06 m, which is 6% uncertainty. Even though the percent uncertainty for the y-intercept is not extremely small, it still makes the calculations of unknown variables (either $h/L$ or $m$) in the equation more accurate when compared to forcing the y-intercept in equation 3 to 0. Eliminating the y-intercept changes the slope of the line dramatically from 22.954 to 33.705, increasing the percent uncertainty of the slope from 3% to 11%. Therefore, it is important to keep both the slope and the y-intercept in the equation for the most accurate prediction.

For the final egg bungee jump, length of the bungee is a variable to be determined when designing a bungee. Equation 3 can be used and written as the following to solve for the length of the bungee cord:

$$Equation 5: L = h/(22.954m + 0.9776)$$

Since there is some amount of uncertainty, there must be experimental errors associated with them. Some errors in measuring the height of the drop might have occurred using the iPad, because many times the images captured at the maximum height of the drop were unclear and tilted at an angle. In addition, some error may be related by the inconsistent knotting of the bungee to the hanging mass, which could slightly change measurements. Random errors such as the elasticity of the cord changing after being stretched multiple times could also affect the accuracies in measurements of the length of the cord and the height of drop.
To improve the accuracy of the experiment and model, it would be useful to collect more data on height of drop so that there would be less variation in the average height values and reduce the impact of random errors. In addition, using longer cord lengths and heavier masses so that a larger height of drop was achieved would be more representative in modeling the egg bungee jump.

V. Conclusion

This experiment helps illuminate the relationship between changing variables for a bungee dropped with a hanging mass attached, namely, the height of drop, length of bungee cord, and mass. We found that there seems to be a linear relationship between the ratio of height of drop to length of bungee (h/L) and mass (m), modeled by \( h/L = 22.954m + 0.9776 \). This equation is useful in predicting h, L, or m when two of the three other variables are known. For the final egg bungee jump, this model can help us predict an accurate length of the bungee cord for the jump. However, it is not known if the same relationship between the variables remains when more than one cord is attached to the mass. This would be interesting to investigate in future experiments.