Introduction:
The restoring force of a spring is the force of the spring on mass \( m \), which acts to restore the mass to its equilibrium position. This restoring force for an ideal spring adheres to Hooke’s Law (Equation 1) and is directly proportional to the change in position of the mass from its equilibrium position, \( x \). This linear relationship produces the constant \( k \), which is known as the spring constant.

\[
F = -kx
\]  

(1)

Where the minus sign tells us that the force acts in the opposite direction of the stretch of the spring, \( F \) tells us the Force a spring exerts on an object in Newtons (Restoring Force), \( k \) is the spring constant in Newton/meters and \( x \) is the distance of the spring from its equilibrium position in meters.

Previous experiments show that our bungee system does not adhere perfectly to Hooke’s Law. However, due to the elastic nature of the bungee cord, we know that the cord does have a restoring force. We will calculate the maximum acceleration of a system to examine the relationship between the maximum acceleration of the system, the restoring force, and bungee cord length using Equation 2.

\[
a_{\text{max}} = \frac{F}{m} - g
\]

(2)

Where \( F \) is the restoring force in Newtons, \( m \) is the mass of the system in kg and \( g \) is the acceleration due to gravity (9.81 m/s²).

We test the relationship of \( a_{\text{max}} \) and the restoring force with respect to various bungee cord lengths by changing the length of the bungee cord. We predict that by increasing the length of the cord, the acceleration of the system will also increase because the system, the “jumper” in a bungee jump, will have more time in free fall before the cord begins to exert a force to “restore” the jumper to its equilibrium position. In general, if the acceleration of a system with a constant mass is greater, then the force of the system should also be greater by Newton’s 2\(^{\text{nd}}\) Law, given in Equation 3, as force and acceleration are directly proportional.

\[
F = ma
\]

(3)

where \( F \) is the force of the system, \( m \) is the mass of the system in kg, and \( a \) is the acceleration of the system in m/s².
The force we will measure is the restoring force of the bungee. We will measure the force exerted on the bungee cord at its maximum acceleration, which will be the restoring force of the system at the farthest displacement $x$.

**Methods:**
To begin this experiment, we hung an analog force sensor at a height of approximately 2 meters from the ground on a stand, which was clamped to our lab table (See Figure 1).

We kept the mass of our system at a constant mass of 150g throughout the experiment to focus on the relationship between cord length and the restoring force and cord length and maximum acceleration. We calibrated the force sensor for each cord length as the mass of the hanging portion of the bungee cord changed slightly depending on the length of the cord. We then tied loops in our bungee cord using a “balloon” style knot. The top loop was hung from the force sensor while the hanging mass system, which consisted of 100g of mass on our 50g mass “skeleton”, was hung from the bottom loop. $L$ refers to the bungee length of our 4 trial groups. Our 4 trials had bungee lengths: .11m, .18m, .27m, and .35m, measured from knot to knot. After calibrating the force sensor to tare for the cord length of each trial, we began recording the force sensor data in Capstone and then dropped our mass system from rest at the zero point to test for the maximum force exerted by the mass. After the first rebound of our system, we stopped recording the force. The force sensor measured the restoring force of the bungee. We proceeded to run 3 trials for each cord length to calculate an appropriate average restoring force.

**Results:**
Firstly, the data we collected on the relationship between cord length and the restoring force, shown in Table 1, is linear by the equation $y=0.55x+3.483$ (See Figure 2) where $y$ is the restoring force and $x$ is the cord length.

| Table 1: |  |
|---|---|---|
| **Cord Length (m, ± 0.01m)** | **Restoring Force (N, ± .001N)** | **Max Acceleration (m/s$^2$, ± .01m/s$^2$)** |
| 0.11 | 3.540 | 13.79 |
| 0.18 | 3.548 | 13.84 |
| 0.27 | 3.596 | 14.16 |
| 0.35 | 3.670 | 14.64 |
The uncertainty for our raw data was given by the ruler’s least count. The uncertainty for the restoring force was calculated while taking the average restoring force using the sum of squares formula in which the raw uncertainty was given by the least count of the force sensor. The uncertainty for maximum acceleration was determined using the Avoidance Method. The percent uncertainty in determining the restoring force we used for each increasing cord length was 0.4%, 1%, 0.3%, and 0.2% respectively. The linear regression analysis for Figure 2 gave a standard error value for our slope of 0.1, and we calculated that our slope has 20% uncertainty.

Figure 2: Restoring Force vs Bungee Cord Length. Graphical representation of the relationship between the restoring force of the system and the length of the bungee cord.

Figure 3: $a_{\text{max}}$ vs L. Graphical representation of the relationship between the maximum acceleration
Secondly, the data we collected, shown in Table 1, shows maximum acceleration increases as bungee cord length increases. The maximum acceleration of the system was calculated using Equation 2. The trend line of our original data of \( a_{\text{max}} \) vs. cord length was polynomial (See Figure 3) but after setting the y-intercept to 0, the relationship was shown to be linear (See Figure 4) and shows as the cord length increased, the maximum acceleration of the system increased as well. The linear relationship is given by the equation \( y = 3.6324x + 13.276 \) where \( y \) is the maximum acceleration of the system and \( x \) is the cord length. Our \( R^2 \) value was 0.92, which means that we can use this equation to predict future outcomes. The linear regression analysis for our data in Figure 4 gave a standard error value for the slope of 0.7, which we used to calculate a 20% uncertainty of our slope in Figure 4.

![Graphical representation of the relationship between the maximum acceleration of the system and the length of the bungee cord.](image)

**Figure 4:** \( a_{\text{max}} \) vs L. Graphical representation of the relationship between the maximum acceleration of the system and the length of the bungee cord.

**Discussion:**

By increasing the bungee cord’s length, the mass of the system (The “jumper” in a bungee jump) has a longer amount of time to fall before the bungee exerts a restorative force on the mass. “The longer free fall time increased the necessary maximum acceleration because the velocity at the beginning of the deceleration was higher, which required longer elongation of the cord, and created a higher restorative acceleration as it was stretched further.”\(^1\) This increase of “free fall” time did in fact increase the maximum acceleration reached by the system (See Table 1) however, the acceleration remained relatively constant.

The bungee cord is not an ideal spring so it does not adhere fully to Hooke’s Law. We determined that the restorative force of the bungee at maximum acceleration of the

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system remained relatively constant, though it does increase slightly. The linear equation we derived to determine the relationship between the restoring force and length of the bungee cord (See Figure 2) shows that there is a nearly negligible change in force due to changing bungee cord length. By having the y-axis run from 0 to 4 we were able to clearly see that the restoring force is nearly constant. If we were to have started our y-axis at 3, then we would not have seen this relationship clearly (Similar to Figure 3). The force remaining constant could be because the k value changes depending on the length of the cord, which was proven in previous experiments. The mass system attached to our elastic bungee cord will behave differently than if it were attached to a spring because force and displacement have a directly proportional relationship for an ideal spring and our bungee cord is not an ideal spring.

In order for the bungee to rebound, the restoring force must be greater than $m_{\text{system}} \times g$. If that condition is met, then we can calculate the maximum acceleration values. The parameters of the bungee challenge are that $a_{\text{max}}$ must be less than three times the acceleration due to gravity ($29.43 \text{m/s}^2$) to avoid damage to the egg. The $a_{\text{max}}$ of our system was 14.64 m/s$^2$, which is approximately 50% of the maximum acceleration our egg can withstand. This being said, more research is required in order for our egg to have a more “thrilling” jump.

Some sources of uncertainty stemmed from the release of our mass system. If our mass system was not released from rest, but rather had some force propulsion directed downwards, the maximum acceleration of the system would have changed. The automated release system provided in the final bungee challenge will eliminate this uncertainty. Incorrect measurement of the bungee cord lengths could result in skewed data which would have altered the relationship between bungee cord length and the restoring force and maximum acceleration. The percent uncertainty of the restoring forces were all less than 1%, which shows that our data for the restoring forces was very precise. The low uncertainty of our force measurements supports our claim that the length of the cord on the restoring force is actually negligible. However, our standard error value for the slope of Figure 2 and Figure 4 was 0.1 and 0.7 respectively, which shows that both our slopes had 20% uncertainty. By setting the y-intercept to 0, our data was subjected to an increased uncertainty. Hooke’s Law gives the restoring force of an ideal spring but we have concluded that the bungee cord is not an ideal spring so we cannot use Hooke’s law to determine any quantitative error.

**Conclusion:**

We predicted that by increasing the length of the bungee cord, the maximum acceleration of the system would also increase because there would be more time for the system to fall. However, the increase in acceleration was nearly negligible as the actual effect of an increasing cord length allowed for minimal increase in maximum acceleration, which is shown in Figure 4 by setting the y-axis to 0.

We predicted that the restoring force would change depending on the length of the bungee cord. However, we discovered that the length of a bungee cord as a minimal, negligible effect on the restorative force of the system, which we did not expect.

In the future, I would focus on how changing the mass of the bungee system affects the restorative force, as this experiment has proved that string length has little to no effect on the restorative force of the system.